A
NEW SYSTEM
OF
ARITHMETIC.

TO WHICH IS ADDED

A GEOGRAPHICAL TABLE.

By JOHN DAVISON,

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AND

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PREFACE.

IT is not only exceedingly uſeful, but very commendable and genteel to be a ſkilful, ready accountant. Thoſe engaged in commerce cannot be too fully acquainted with arithmetic; for to tranſact buſineſs with eaſe, diſpatch, and propriety, they muſt be extremely expert in whatever relates to accounts.

If this ſcience was uſeful only to the commercial part of mankind, the importance of it would be very great, eſpecially in a kingdom like this, where ſuch a large proportion of the inhabitants are employed in trade. But the utility of Arithmetic appears much greater when it is obſerved that there is no profeſſion, employment, or ſtation, however exalted, but where a competent knowledge at leaſt, of numbers is abſolutely neceſſary.

It is poſſible for a perſon to be ſo complete an accountant, that while he is writing a line expreſſing any quantity of goods, at a given price, he can compute the amount, mentally, and conſequently write the whole line, including the value, in the ſame time that he could write it, if the amount had been previouſly known. For inſtance, if 144 yards of muſlin at 5*s.* 3*d.* per yard, and the value are to be expreſſed in writing, I am convinced that a perſon may be ſo ſkilful in numbers as to find the value mentally, while he is writing the other part of the line.

One of the principal deſigns of this publication is to form the perfect accountant, who can perform all his computations in the ſhorteſt and moſt correct manner. The ſyſtem is divided into five books; in the firſt the fundamental rules are demonſtrated, and the nature of numbers illuſtrated. Vulgar and Decimal Fractions are included in the ſecond book. One ignorant of theſe will often be obliged to perform tedious operations, where Fractions could readily effect the deſired purpoſe.

Practice and the Golden Rule are contained in the third book; with every part of which the pupil muſt be fully acquainted, for theſe are the grand rules which properly conſtitute the perfect accountant, thoſe contained in the two preceding books, being, with reſpect to him, little more than preparations for theſe two excellent rules; and what is contained in the fourth book is no more than an application of the third.

The fourth book, as has juſt now been obſerved, is the application of Practice and the Golden Rule to the other mercantile parts of arithmetic, as Loſs and Gain, Intereſt, Exchange, &c. The rules in the fifth book are not applicable to mercantile purpoſes, but uſeful in the mathematics, theſe are Involution, Evolution, Progreſſion, and Poſition. The accountant, unleſs he is alſo intended for a mathematician, has no occaſion for the fifth book.

This ſyſtem is intended to prepare youth for entering on the mathematics, as well as to form the perfect accountant; for this purpoſe all the five Books are neceſſary.

PREFACE.

I am very far from preſuming to dictate to any gentleman employed in teaching, concerning the manner of uſing this ſyſtem, I only beg leave to mention the method that I propoſe obſerving in my ſchool. If it coincides with the opinion of ſome, and furniſhes hints for others, it will give me great pleaſure.

Some weeks before the ſcholar actually begins arithmetic at ſchool, it is intended he ſhall have this ſyſtem to get off the characters, abbreviations, tables of weights and meaſures, pence table, and multiplication table.

When he can repeat the tables readily, he is to begin arithmetic in a book of fine writing paper, and inſert all that is contained in the firſt page of this ſyſtem, that he may afterwards find no difficulty in making the characters. Numerical expreſſion is then to be perfectly underſtood, after which he is to proceed to the fundamental rules, and be made thoroughly ſenſible of their nature, and the manner of performing them.

I do not intend that my ſcholars ſhall write any of the rules into their books, only get them off, and be made to underſtand them well. The titles of the rules they are to write at the top of the pages, and they are likewiſe to write the examples and their ſolutions as neatly and elegantly as poſſible. They will look back on what they have gone through with pleaſure, if done properly, be encouraged to proceed with attention, and conſequently improve as rapidly in writing as in arithmetic.

As ſoon as any of the ſcholars begin Vulgar Fractions, I deſign to inform them that they are to apply themſelves, at every opportunity, to the rules and tables of Practice; that when they are done with Decimals they may be qualified to begin the examples in Practice.

When any of the ſcholars are to proceed to a courſe of mathematics, I propoſe that they ſhall begin to read my Complete Courſe of Mathematics, at the ſame time that they begin the fifth book of this ſyſtem, whoſe rules are derived from Algebra, by which means they will be able to proceed regularly to the problems when arithmetic is finiſhed. If they are only to be inſtructed in ſome of the practical parts of the mathethematics, it is propoſed that they ſhall, at every opportunity, while in the fifth book, be ſhewn the manner of raiſing perpendiculars, and other parts of practical geometry.

One afternoon is appointed every week for examining the pupils in the rules they have gone through.

Every part of this ſyſtem it is hoped will be allowed to be plain, conciſe, and original. If in general it be honored with approbation, I ſhall be extremely happy. When arithmetic is readily acquired, young gentlemen will have time to proceed to mathematical ſtudies, which will enlarge the mind, and be a rational entertainment during life.*

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The Geographical Table which is added will be particularly uſeful to the merchant and mariner, and cannot fail of being agreeable to every one.

JOHN DAVISON.

Exeter Court, Strand,
December, 1779.

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* See the Introduction to the Courſe of Mathematics.

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E R R A T A.

In the table of ale and beer meaſure, page 8, for 52 read 51.—For 671 in the laſt example, page 32, read 1675,—In the fourth example, page 40, inſtead of 7324 read 7328.—For 108 read 15, in the firſt line of page 71.—In the laſt example but one, page 110, for 52 read 32.—For $\frac{3}{8}$ read $\frac{7}{8}$, example 2d, page 120.——In the ſolution, page 176, for the nineteenth read the fourteenth.

ARITH-

ARITHMETIC.

BOOK I.

Characters, Abbreviations, Notation, Tables, and the Fundamental Rules.

CHARACTERS.

Signs.	Significations.	Signs.	Significations.
+	More, or add.	∷	So is.
—	Leſs, or ſubtract.	$\overset{..}{-}$	Proportion.
×	Multiply, multiplied.	$\sqrt{}$	Square Root.
÷	Divide, or divided.	$\sqrt[4]{}$	Biquadrate Root.
=	Equal.	∴	Therefore.
:	Is to. Alſo, to.		

ABBREVIATIONS.

Short.	Meaning.	Short.	Meaning,
a	Acre, or Acres	*k*	Kilderkin
b	Buſhels	*l*	Pounds of Money
c	Hundred Weight	*lb*	Pounds in Weight
d	Pence	*m*	Miles
dr	Drams	*oz*	Ounces
dwt	Pennyweights	*p*	Pecks—Perches
e	Ells	*qr*	Quarters
f	Feet—Furlongs	*r*	Roods
gr	Grains	*s*	Shillings
h	Hours	*ſc*	Scruples
hhd	Hogſheads	*t*	Tuns
i	Inches	*yd*	Yards

The ſame abbreviation is uſed for the ſingular as for the plural; thus *l* ſtands for either *pound* or *pounds*; *r* for either *rood* or *roods*, &c.—The abbreviations may be either in capitals or ſmall letters.

Some of theſe letters are alſo occaſionally taken to repreſent other words than thoſe they are ſet againſt. But their ſignifications are then ſo obvious, that it would have been improper to have given them in the table. Thus, in mentioning time, it is plain that *d* ſtands for *days*, and *s* for *ſeconds*.

NOTATION,

OR

NUMERICAL EXPRESSION.

ALL numbers are expreſſed by one or more of theſe characters or figures, viz. 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. When two of them are placed together, the firſt, on the right, expreſſes ſo many units or ones, and the other ſo many tens. Thus 34 is the notation or numerical expreſſion for thirty four; or three tens and four units.

When three figures are placed in a line, the firſt on the right is units, and the ſecond tens, as before, and the third hundreds; for inſtance 745 ſtand for ſeven hundred and forty five. That is, ſeven hundreds, 4 tens and 5 units.

When there are four figures in a line, the firſt is units, the ſecond tens, and the third hundreds, as before, and the fourth is thouſands. Thus 8634 is the expreſſion for eight thouſand, ſix hundred and thirty-four.

In General. The value of a figure is ten times greater for every removal from the place of unity. The following line, with its explanation, will ſerve as a precedent for writing numerically any number which can occur, either in queſtions of real uſe or curioſity.

Trillions						Billions						Hundreds of thouſands of millions	Tens of thouſands of millions	Thouſands of millions	Hundreds of millions	Tens of millions	Millions	Hundreds of thouſands	Tens of thouſands	Thouſands	Hundreds	Tens	Units
9	8	7	3	6	7	2	8	7	3	6	9	5	4	9	2	7	6	5	8	7	6	7	8

Or 987 thouſand 367 trillions, 287 thouſand 369 billions, 549 thouſand 276 millions, 587 thouſand 678.

Set down in figures, Five hundred and thirty-eight.

Anſwer 538.

Set down in figures, Six hundred and twenty ſeven.

Write in figures, Nine thouſand, ſeven hundred and twelve.

Anſwer 9712.

Write in figures, Four thouſand, eight hundred and ſeventy-two.

Expreſs in figures, Seven thouſand, and four hundred.

Anſwer 7400.

Expreſs in figures, Eight thouſand and two hundred.

Set down numerically, Fourteen thouſand eight hundred and twenty-nine.

Anſwer 14829.

Set down numerically, Twenty-five thouſand, ſeven hundred and forty-four.

Write numerically, Thirty-two thouſand and five hundred.

Anſwer 32500.

Write numerically, Seventy-two thouſand and two hundred.

Expreſs numerically, Forty-two thouſand and twenty-ſeven.

Anſwer 42027.

Expreſs numerically, Fifty-ſeven thouſand and eighty-nine.

Write in figures, Sixty thouſand and five.

Anſwer 60005.

Write in figures, Eighty thouſand and four.

Expreſs in figures, Four hundred and thirty-nine thouſand, ſix hundred and fifty-three.

Anſwer 439653.

Expreſs in figures, Seven hundred and forty-nine thouſand, nine hundred and two.

Set down numerically, One hundred thouſand.

Anſwer 100000.

Write numerically, Sixty millions.

Expreſs numerically, Six hundred and twenty-four millions, nine hundred and ſeventy-two thouſand, ſix hundred and ſeventy-eight.

Anſwer 624972678.

Expreſs numerically, Seven hundred and eighty-five billions.

Anſwer 785000000000000.

When a number is all of one kind or denomination, as all pounds; or all yards; or all acres, &c. it is a whole number. All the foregoing numbers are whole numbers.

But when there are different denominations, as pounds, ſhillings and pence, it is a compound. The remaining examples ſhew the notation of compounds.

Set down in figures, Seven hundred and thirty-ſix pounds, thirteen ſhillings, and ten pence.

Anſwer 736*l.* 13*s.* 10*d.*

Set down in figures, Seventeen thouſand, eight hundred and two pounds, one ſhilling and ſix pence.

Write numerically, Ten tuns, eleven hundred weight, one quarter, two pounds, thirteen ounces, and fourteen drams.

Anſwer 10*t.* 11*c.* 1*q.* 2*lb.* 13*oz.* 14*dr.*

Write numerically, Six hundred and ninety-two acres, three roods, and thirty-ſix perches.

Anſwer 692*a.* 3*r.* 36*p.*

Expreſs numerically, Fifty acres and twenty perches.

TABLES
OF
MONEY, WEIGHTS and MEASURES.

MONEY.

4	farthings	=	1 penny.
12	pence	=	1 ſhilling.
20	ſhillings	=	1 pound ſterling.

TROY WEIGHT.

24	grains	=	1 pennyweight
20	pennyweights	=	1 ounce
12	ounces	=	1 pound.

By this weight, gold, ſilver, jewels and liquors are weighed.

AVOIRDUPOISE WEIGHT.

16	drams	=	1 ounce
16	ounces	=	1 pound
28	pounds	=	1 quarter
4	quarters	=	1 hundred weight
20	hundred weight	=	1 tun.

This is the general weight for buying and ſelling.

APOTHECARIES WEIGHT.

20	grains	=	1 ſcruple
3	ſcruples	=	1 dram
8	drams	=	1 ounce
12	ounces	=	1 pound.

The apothecaries only compound their medicines by this weight, for they buy and ſell by avoirdupoiſe.

A pound apothecaries weight is the ſame as a pound Troy weight.

WOOL WEIGHT.

7 pounds	=	1 clove
2 cloves		1 ſtone
2 ſtones		1 todd
$6\frac{1}{2}$* todds		1 wey
2 weys		1 ſack
12 ſacks		1 laſt.

* Obſerve that $\frac{1}{2}$ ſtands for one-half; alſo that $\frac{1}{4}$ ſtands for one quarter; and $\frac{3}{4}$ for three quarters.

LAND MEASURE.

40 poles, or perches	=	1 rood
4 roods		1 acre.

LONG MEASURE.

12 inches	=	1 foot
3 feet		1 yard
$5\frac{1}{2}$ yards		1 pole
40 poles		1 furlong
8 furlongs		1 mile
3 miles		1 league.

CLOTH MEASURE.

$2\frac{1}{2}$ inches	=	1 nail
4 nails		1 quarter
4 quarters		1 yard
3 quarters		1 ell Flemiſh
5 quarters		1 ell Engliſh
6 quarters		1 ell French.

⁂

DRY MEASURE.

2 pints	=	1 quart
2 quarts	=	1 pottle
2 pottles, or 8 pints	=	1 gallon
2 gallons	=	1 peck
4 pecks, or 8 gallons	=	1 buſhel
8 buſhels	=	1 quarter
5 quarters	=	1 wey
2 weys	=	1 laſt.

This meaſure is for corn, and all other dry goods, except coals. Three buſhels of coals make 1 ſack; 12 ſacks or 36 buſhels 1 chaldron; and 21 chaldron 1 ſcore.

WINE MEASURE.

8 pints	=	1 gallon
42 gallons	=	1 tierce
$1\frac{1}{2}$ tierce, or 63 gallons	=	1 hogſhead
2 hogſheads	=	1 pipe or butt
2 pipes, or 4 hhds	=	1 tun.

Rum, brandy, cordial waters, &c. are alſo bought and ſold by wine meaſure.

ALE and BEER MEASURE.

8 pints	=	1 gallon
$8\frac{1}{2}$ gallons	=	1 firkin
2 firkins	=	1 kilderkin
2 kilderkins	=	1 barrel
$1\frac{1}{2}$ barrel, or 52 gallons	=	1 hogſhead
2 hogſheads	=	1 butt.

In all parts of the country, the above table is uſed for both ale and beer; but in London 8 gallons are 1 firkin of ale, and 9 gallons 1 firkin of beer.

TIME.

60 ſeconds	=	1 minute
60 minutes	=	1 hour
24 hours	=	1 day
7 days	=	1 week
4 weeks	=	1 lunar month
13 months, 1 day	=	1 common year.

The TWELVE SOLAR, *or* KALENDAR MONTHS, *are*

	Days.		Days.
January	31	July	31
February	28	Auguſt	31
March	31	September	30
April	30	October	31
May	31	November	30
June	30	December	31

Therefore every common year has 365 days. A leap-year, which is every fourth year, contains 366 days, February having then 29 days. The preſent is the third after leap-year, and therefore the next is leap-year.

* * *

FUNDAMENTAL RULES.

ADDITION, Multiplication, Subtraction, and Diviſion, are the fundamental operations of arithmetic. By a judicious application of theſe, all queſtions in this ſcience are anſwered.

ADDITION.

BY Addition, the total, or amount of ſeveral particular ſums is known. For inſtance, if the three numbers 752; 936 and 784 are given, the operation whereby their total or amount is known to be 2472, is addition.

OF WHOLE NUMBERS.

THE ſeveral numbers being properly placed under each other, viz. units under units, tens under tens, &c. begin at the line of units, add them; place down what is above 10, or 20, or 30, &c. and for every 10 in it, add 1 to the next line. Do the ſame with every line. Set down all that the laſt line of the queſtion comes to.

It is eaſy to conceive the reaſon of the above rule. By the nature of numbers, page 3, ten in the line of units is equal to 1 in the line of tens; 10 in the line of tens is equal to 1 in the line of hundreds, &c.

In the three following examples, the numbers are already properly placed, and to the firſt of them the anſwer is given.

£	£	Yd.
643	5384	473694
375	6675	748632
954	7493	74964
862	6469	826
2834		

The firſt queſtion. Add 2 to 4, the ſum is 6; 6 and 5 are 11; 11 and 3 are 14, which is 4 above 10, therefore ſet down 4 and take 1 to the next line—Add 1 to 6, the ſum is 7; 7 and 5 are 12; 12 and 7 are 19, and 4 makes 23, which is 3 above 20, therefore ſet down 3, and becauſe there are 2 tens in 23, take 2 to the next line—Add 2 to 8, the ſum is 10; 10 and 9 are 19, and 3 are 22, and 6 are 28; which being the amount of the laſt line, muſt be all ſet down.

The three following examples will exerciſe the learner in placing the numbers in a proper manner.

A gentleman has in one county 8324 acres, in another 634, and in another, thirty thouſand, nine hundred and one. How many has he in all?

A.
8324
634
30901

Anſwer 39859 *acres.*

A merchant received from A 746*l.* from B 2045*l.* from C 494*l.* from D ſix hundred pounds, and from E 36*l.* How much did he receive in all?

Anſwer 3921*l.*

The father of two ſons and two daughters, gave them as follows. To the eldeſt ſon, 17008*l.* to the ſecond, five thouſand and two hundred; to the eldeſt daughter, 2993*l.* and to the ſecond, 974*l.* Required the whole ſum given.

Anſwer 26175*l.*

ADDITION

OF

MONEY.

BEGIN at the farthings;* add them, and for every 4 add 1 to the line of pence. For every 12 pence, take 1 to the ſhillings; and for every 20 ſhillings, take 1 to the firſt line of the pounds. The pounds are added by the inſtructions given in whole numbers, by carrying 1 for 10.

The following table ſhews the ſhillings which are in any number of pence, and alſo on the contrary, the pence which are in any number of ſhillings.

Explanation. Look in one of the columns under *d* for the pence, and againſt it, on the right, is the ſhillings; thus, 70 pence is 5*s*. 10*d*; and 160 pence is 13*s*. 4*d*.

If ſhillings are given to find the pence, look for the ſhillings, and on the left is the pence; for inſtance, 15*s*. is 180*d*.

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[*] One farthing is written $\frac{1}{4}$; that is one-fourth of a penny. Two farthings are expreſſed $\frac{1}{2}$, or one-half of a penny; and three farthings are written $\frac{3}{4}$, or three-quarters of a penny.

PENCE TABLE.

d.		s.	d.	d.		s.	d.	d.		l.	s.	d.
12	is	1	0	96	is	8	0	180	is		15	0
20	....	1	8	100	....	8	4	190	....		15	10
24	....	2	0	108	....	9	0	192	....		16	0
30	....	2	6	110	....	9	2	200	....		16	8
36	....	3	0	120	....	10	0	204	....		17	0
40	....	3	4	130	....	10	10	210	....		17	6
48	....	4	0	132	....	11	0	216	....		18	0
50	....	4	2	140	....	11	8	220	....		18	4
60	....	5	0	144	....	12	0	228	....		19	0
70	....	5	10	150	....	12	6	230	....		19	2
72	....	6	0	156	....	13	0	240	....	1	0	0
80	....	6	8	160	....	13	4	252	....	1	1	0
84	....	7	0	168	....	14	0					
90	....	7	6	170	....	14	2					

l.	s.	d.
43276	10	10½
87268	14	7¼
54727	6	9¾
185272	12	3½

l.	s.	d.
47284	11	6½
64369	6	4¼
7264	4	2½

78492

l.	*s.*	*d.*	*l.*	*s.*	*d.*
78492	17	11¼	4368	19	4¾
67487	10	6½	746	11	2¼
90200	6	8	849	10	7¾
4728	13	0¼	95	4	8½

The firſt example of money explained—Add 3 farthings to 1 farthing, the ſum is 4, and 2 farthings make 6 farthings, which is ½ above 1 penny, therefore ſet down ½ and take 1 to the line of pence; which then comes to 27. In 27*d* there is 2*s* 3*d*; ſet down 3, and add 2 ſhillings to the line of ſhillings, which then comes to 32. In 32 ſhillings there are 1*l.* 12*s*; ſet down 12 and add 1 to the firſt line of pounds. The pounds are added by the rule for whole numbers.

A gentleman paid the following bills, viz. the mercer's 69*l.* 11*s.* 7*d.*; the linen-draper's 13*l.* 2*s.* 7¾*d.*; the wine-merchant's 75*l.* 10*s.* 9½*d.*; and the goldſmith's 354*l.* 13*s.* 10*d.* How much did he pay in all?

OPERATION.

	l.	*s.*	*d.*
The mercer	69	11	7
Linen-draper	13	2	7¾
Wine-merchant	75	10	9½
Goldſmith	354	13	10
	£512	18	10¼

The effects of a merchant are, in goods 5724*l* 10*s* 9½*d*; in notes 4175*l*; the balance of book debts 7612*l* 11*s* 10½*d*; bonds one thouſand pounds; furniture 912*l* 11*s* 2½*d*; and in caſh, two thouſand and forty-ſeven pounds, four ſhillings and three farthings.——Required his whole eſtate?

Anſwer 21471*l* 17*s* 11¼*d*

A nobleman has, in the county of Northumberland, an eſtate worth twenty-three thouſand, ſix hundred and eighty-nine pounds, thirteen ſhillings, and ten-pence, yearly: In Yorkſhire, an eſtate worth five thouſand and ſeven pounds: And in Kent, ſixteen thouſand, eight hundred and two pounds, four ſhillings, and eleven-pence. Required his annual revenue?

A farmer paid his rent, 296*l*. 10*s*.—ſervants wages, 41*l*. 10*s*. 6d.—for a ſaddle, two guineas;—a bridle, 3*s*. 6*d*.—a baſket, 9*d*.—a hat, fifteen ſhillings;—and for ten oxen, one hundred pounds. How much did he pay in the whole?

Add a guinea, half-a-guinea, a crown, half-a-crown, a ſhilling, and a ſix-pence, into one ſum.

A houſekeeper paid, for beef, 15*s*. 9½*d*.—for mutton, 5*s*. 2*d*.—two turkies, half-a-guinea;—chickens, 5*s*. 1½*d*.—apples, 1*s*. 4½*d*.—turnips, 3½*d*.—and for carriage, 2*d*. How much in all?

Paid for a pair of globes, ſix guineas;—other inſtruments, 7*l*. 11*s*. 8*d*.—Daviſon's Complete Courſe of Mathematics, 3*l*.—chairs, 2*l*. 1*s*. 7*d*.—muſlin, 11*s*. 9*d*.—coffee, 6*s*. 9*d*.—cinnamon, 9½*d*.—ſilver ſpoons, 3*l*. 1*s*. 8*d*.—and chocolate, 7*s*. 3½*d*. Required the whole ſum paid?

ADDITION OF WEIGHTS AND MEASURES.

TROY WEIGHT. Add the grains, and for every 24* grains add 1 to the penny-weights, and ſet down the odd grains, if any, under grains. For every 20 penny-weights add 1 to the ounces, ſetting down the odd penny-weights, if any, under penny-weights. For every 12 ounces add 1 to the lbs; which being whole numbers are added accordingly.

In a ſimilar manner, the other weights, and the meaſures are added, regard being had to their reſpective tables.

An EXAMPLE *in Troy Weight.*

lb.	*oz.*	*dwt.*	*gr.*
274	8	14	16
467	6	10	10
86	3	18	12

An EXAMPLE *in Apothecaries Weight.*

lb.	*oz.*	*dr.*	*ſc.*	*gr.*
67	8	7	2	10
76	4	4	1	16
48	6	4	2	8

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* See the table of Troy weight, page 7.

EXAMPLES *in Avoirdupoiſe Weight.*

t.	*c.*	*q.*	*lb.*	*oz.*	*dr.*
634	17	1	17	10	8
76	8	2	8	8	10
84	6	3	8	0	0
	8	2	5	6	4
796	1	1	11	9	6

A merchant has in one warehouſe 4*t.* 3*c.* 2*qr.* of goods; in another, 13*c.* 2*qr.* 14*lb.* and in another, 4*t.* 11*c.* 3*qr.* 14*lb.* 9*oz.* 6*dr.* Required the total?

An EXAMPLE *in Long Meaſure.*

m.	*f.*	*p.*	*yd.*	*f.*	*i.*
632	4	21	4	2	10
746	7	8	3	2	11
4784	6	3	5	1	8

MULTIPLICATION.

WHEN ſeveral equal numbers, as 635, 635, 635, 635, 635, 635, and 635 are given, the operation, whereby their amount is moſt readily known to be 4445, is called Multiplication.

Multiplication is therefore an expeditious method of adding a multitude of equal numbers.

Amount in Addition is called product in Multiplication.

The following TABLE, which it is abſolutely neceſſary to know off-book, ſhews the product of any two nnmbers, each under 13.

	A												
B	1	2	3	4	5	6	7	8	9	10	11	12	B
	2	4	6	8	10	12	14	16	18	20	22	24	
	3	6	9	12	15	18	21	24	27	30	33	36	
	4	8	12	16	20	24	28	32	36	40	44	48	
	5	10	15	20	25	30	35	40	45	50	55	60	
	6	12	18	24	30	36	42	48	54	60	66	72	
	7	14	21	28	35	42	49	56	63	70	77	84	
	8	16	24	32	40	48	56	64	72	80	88	96	
	9	18	27	36	45	54	63	72	81	90	99	108	
	10	20	30	40	50	60	70	80	90	100	110	120	
	11	22	33	44	55	66	77	88	99	110	121	132	
	12	24	36	48	60	72	84	96	108	120	132	144	

The uſe of the table.—Let it be required to find the product of 9 times 8;—in the line A, look for 9, and in the line B look for 8. Proceed forward from 9, and downward from 8, and 72 will appear to be their product.

Again, let the product of 12 times 7 be required.—In the line A look for 12, and in the line B for 7. Proceed from 12 and 7, and 84 is found for their product.

The ſcholar being very ready at repeating the table without the book, he muſt proceed to the multiplication of numbers in general, by the following rules:

I. When the multiplier is leſs than 13, multiply it into the firſt figure of the multiplicand, and ſet down the odd units.

Multiply the ſecond figure of the multiplicand, and to the product add 1 for every 10 in the former product. Set down the overplus of the ſum.

Multiply the third figure of the multiplicand, and to the product add 1 for every 10 contained in the preceding ſum. In the ſame manner, multiply every figure of the multiplicand. The laſt ſum is to be ſet down entire.

EXAMPLES.

Multiplicand	4893	Multiplicand	684879
Multiplier	6	Multiplier	7
Product	29358	Product	4794153

The firſt example. Multiply 6 by 3, the product is 18, which contains 1 ten, and 8 over. Set down 8.

Multiply 6 by 9, the product is 54, to which add 1 for the ten in the former product, the ſum is 55. Set down 5.

Multiply 6 by 8, the product is 48, to which add 5, the tens in the preceding ſum 55, and the ſum is 53. Set down 3.

Multiply 6 by 4, the product is 24, to which add 5, the tens in 53, and the ſum is 29; to be ſet down entire, as it is the laſt. Therefore 6 times 4893 is 29358.

The firſt product is units, the ſecond tens, the third hundreds, the fourth thouſands, &c. and by the nature of numbers already explained, 10 in the units make 1 ten; 10 in the place of tens, make 1 hundred; 10 hundreds make 1 thouſand, &c. hence the reaſon of adding 1 for every 10 throughout multiplication.

More EXAMPLES.

Multiply	320686	Multiply	867869
By	11	By	12

II. When the multiplier is any greater number than 12,

Multiply the firſt figure of the multiplier, into every figure of the multiplicand, by Rule I.

By the ſame Rule, multiply the ſecond figure of the multiplier into every figure of the multiplicand. The firſt figure of this line muſt be placed under the ſecond figure of the former line.

Multiply, by the ſame Rule, the other figures of the multiplier, if there are any more, into every figure of the multiplicand.

The firſt figure of each line muſt be placed under the ſecond of the line immediately preceding it, and thereby by tens will be under tens, hundreds under hundreds, &c. For the firſt figure of the multiplier being units, the firſt product is units. The ſecond figure of the multiplier being tens, the firſt product is tens. The third figure of the multiplier being hundreds, the firſt product is hundreds, &c.

Add all the lines, and the ſum is the product required.

EXAMPLES.

Multiply 675364	Multiply 9487968
By 423	By 652
2026092	18975936
1350728	47439840
2701456	56927808
285678972	6186155136

The firſt example. The multiplicand 675364 by 3 produces 2026092. The multiplicand by 2 tens or 20 produces 13507280, for a cypher is underſtood to ſupply the units place. The multiplicand by 4 hundreds or 400 is 270145600, for 2 cyphers are underſtood to ſupply the places of tens and units. The ſum of theſe products 285678972 is evidently the product of 675364 by 423

More EXAMPLES.

The product of 8369546 by 6748 is 56477696408.

The product of 730852 by 56249 is 41109694148.

Multiply 612 by 54, and the product by 32.

When there are cyphers annexed to the multiplier, multiply by the other figure or figures as before, and join the cyphers to the product.

EXAMPLES.

Multiply	6532	Multiply	80542
By	2900	By	56000
	58788		483252
	13064		402710
	18942800		4510352000

The firſt product by 9 is evidently hundreds, and therefore two cyphers are to be joined to make it ſo. In the ſecond example, the firſt product by 6, is thouſands, which makes it neceſſary to place three cyphers to make it ſo; conſequently in general, the cyphers are to be joined to the other product.

If there are cyphers annexed to both multiplicand and multiplier, the other figures are to be multiplied as before, and all the cyphers joined to the product.

EXAMPLES.

Multiplicand	87360	Multiplicand	64300
Multiplier	72000	Multiplier	750000
	17472		3215
	61152		4501
	6289920000		48225000000

The firſt product by 2, is tens of thouſands, for it is thouſands multiplied into tens, and therefore four cyphers are to be joined that it may be ſo. In the ſecond example, the firſt product by 5 is millions for it is tens of thouſands by hundreds, and therefore ſix cyphers are neceſſary, conſequently in general, the cyphers of both multiplicand and multiplier are to be joined to the product.

When there are cyphers amongſt the figures of the multipler, proceed in every reſpect as if there were none, only for every cypher, begin the next line one figure more to the left.

EXAMPLES.

Multiply	37428	Multiply	764240
By	4032	By	160080
	74856		611392
	112284		458544
	149712		76424
	150909696		122339539200

The ſituation of the 4 in the multiplier of the firſt example, is one hundred times greater than the ſituation of the 3 in the ſame example, which is the reaſon that the third line of the multiplication begins two figures more to the left than the line produced by 3.

The ſituation of the 6 in the ſecond example is one thouſand times greater than the ſituation of the 8 in the ſame example, hence the reaſon that the ſecond line of the multiplication begins three figures more to the left than the firſt line.

That the ſcholar may be very ready at ſo excellent an operation as multiplication, the following examples are added, as farther exerciſes in all the caſes which can poſſibly occur.

The product of 76543 by 625 is 47839375.

The product of 8590796 by 669 is required?

The product of 6723 by 12000 is 80676000.

The product of 3125 by 3200 is required?

The product of 62500 by 4800 is 300000000.

The product of 288000 by 3420 is required?

The product of 56967 by 608 is 34635936.

The product of 6028400 by 700600 is required?

To know if a queſtion is right, perform the operation a ſecond time, without looking at the firſt. If the products agree, both operations are right.

Another method of proving multiplication, is to change the multiplier into the multiplicand, and the multiplicand into the multiplier, and in that ſtate to multiply them.

For the product is the ſame whether the greater or leſſer number is conſidered as the multiplicand. Conceive one of the numbers expreſſed by the units of which it conſiſts, and this line of units to be as often repeated as there are units in the other number. The ſum of all theſe units is the product, and it is evidently immaterial which of the lines is conſidered as the multiplicand.

Let *a*, *a*, *a*, &c. from A to B, repreſent the units of one of the numbers; and *a*, *a*, *a*, &c. from A to C, repreſent the units of the other, it is exceedingly plain, that the product, that is the ſum of all the units, will be the ſame, whether A B or A C be called the multiplicand.

A
a, *a*, *a*, *a*, *a*, *a*, *a*, *a*, *a*, *a*, *a*, *a*, B
a, *a*, *a*, *a*, *a*, *a*, *a*, *a*, *a*, *a*, *a*, *a*,
a, *a*, *a*, *a*, *a*, *a*, *a*, *a*, *a*, *a*, *a*, *a*,
a, *a*, *a*, *a*, *a*, *a*, *a*, *a*, *a*, *a*, *a*, *a*.
C

MULTIPLICATION OF MONEY.

WHEN the multiplier is under 13, having placed it under the leaſt denomination of the money, multiply it into every denomination. For every 4 in the product of farthings, add 1 to the product of pence. For every 12 in the amount of the pence, add 1 to the product of the ſhillings; and for every 20 in the amount of the ſhillings, add 1 to the firſt product of the pounds. The pounds, if any, being whole numbers, are to be multiplied by adding 1 for every 10. The odd farthings are to be ſet under the farthings; the odd pence under the pence; and the odd ſhillings under the ſhillings.

EXAMPLES.

EXAMPLES.

How much do 7 acres of land come to, at 23*l.* 9*s.* 4½*d.* each?

l.	*s.*	*d.*
23	9	4½
		7
£164	5	7½

Required the value of 8 yards at 9*s.* 11¼*d.*?

Anſwer 3*l.* 19*s.* 6*d.*

Required the value of 10 yards at 6*s.* 8*d.* per yard?

Anſwer 3*l.* 6*s.* 8*d.*

How much do 11 gallons of wine come to, at 12*s.* 7¼*d.* per gallon?

	s.	*d.*
	12	7¼
		11
£6	18	7¾

Required the value of 9 yards, at 10*s.* 7½*d.* each?

Anſwer 4*l.* 15*s.* 7½*d.*

Required the value of 12 pounds of tea, at 7*s.* 6*d.* per lb.?

Anſwer 4*l.* 10*s.*

When the multiplier is above 12, find two numbers whoſe product is equal to it. Multiply the given price by either of them, and the reſult by the other.

Required the value of 24 yards, at 4*s.* 7½*d.* each?

It is eaſy to perceive that *6 × 4 produces 24; hence the annexed operation.

		s.	*d.*
		4	7½
			6
	1	7	9
			4
Anſwer	£6	11	0

* The 6 and 4 are called the component parts of 24.

The firſt reſult 1*l.* 7*s.* 9*d.* is the value of 6 yards, and therefore 4 times that ſum is evidently the value of 24 yards.

Required the value of 21 gallons, at 9*s.* 3*d.* per gallon?

The component parts of 21 are 7 and 3.

		s.	*d.*
		9	3
			7
	3	4	9
			3
Anſwer	£9	14	3

Required the value of 45 yards, at 1*l.* 2*s.* 3*d.* per yd.?

Anſwer 50*l.* 1*s.* 3*d.*

Required the value of 48 yards of broad cloth, at *17*s.* 6*d.* per yard?

Anſwer 42*l.*

How much do 72 yards come to, at 6*s.* 5$\frac{1}{4}$*d.* per yd.?

Anſwer 23*l.* 3*s.* 6*d.*

Required the value of 110 gallons, at 9*s.* 3*d.* each?

Anſwer 50*l.* 17*s.* 6*d.*

Required the value of 66 gallons, at 4*s.* 8$\frac{1}{2}$*d.* per gallon?

Anſwer 15*l.* 10*s.* 9*d.*

How much do 84 hundred weight come to, at 7*l.* 3*s.* 9*d.* per cwt.?

Anſwer 603*l.* 15*s.*

Required the value of 144 lbs. of tea, at 13*s.* 4*d.* per lb.?

Anſwer 96 *l.*

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* The ſhillings being above 12, multiply thus: Eight times 7 is 56, and 8 times 10 is 80, for 1 in the ſecond place is 10. The ſum of 56 and 80 is 136*s.*—Or 8 times 7 is 56, and 80 is 136; for adding a cypher to the multiplier, is the ſame as multiplying it into 10.

If the multiplier has no component parts, find the value for the next lower number which has ſuch parts, and add the value of the remaining quantity.

How much do 29 yards come to, at 7*s.* $4\frac{1}{2}$*d.* per yd.?

There are no two numbers whoſe product is 29, that is, 29 has no component parts, but 28 has the components 7 and 4.

		s.	*d.*
		7	$4\frac{1}{2}$
			4
	1	9	6
			7
Value of 28yds.	10	6	6
Do. of 1 yd. add		7	$4\frac{1}{2}$
Anſwer £	10	13	$10\frac{1}{2}$

How much do $68\frac{1}{2}$ yards come to, at 8*s.* 10*d.* per yd.?

	l.	*s.*	*d.*
66 yards	29	3	0
2 yards		17	8
$\frac{1}{2}$ yard		4	5
Anſwer £	30	5	1

How much do 38 yards come to, at 9*s.* 10*d.* per yd.?

The neareſt number to 38, which has component parts, is 36.

		s.	*d.*
		9	10
			6
	2	19	0
			6
36 yards	17	14	0
For 2 yds. add		19	8
Anſwer £	18	13	8

Required the value of 135 pair, at 2*l.* 13*s.* $9\frac{1}{4}$*d.* each pair?

How much do $113\frac{1}{4}$ yds come to, at 16*s.* 9*d.* each?

Anſwer 94*l.* 16*s.* $11\frac{1}{4}$*d.*

Required the value of $31\frac{3}{4}$ dozen, at 3*l.* 12*s.* 8*d.* per dozen?

Being well acquainted with the multiplication of money, it will be eaſy to multiply any other compound.

EXAMPLES.

How much do 9 parcels weigh, each weighing 3*qr.* 10*lb.* 8*oz.* 12*dr.*?

	c.	*qr.*	*lb.*	*oz.*	*dr.*
Anſwer ..	7	2	10	14	12

How many acres are in 16 fields, each containing 13*a.* 3*r.* 16*p.*?

	a.	*r.*	*p.*
Anſwer	221	2	16

SUBTRACTION

OF

WHOLE NUMBERS.

BY ſubtraction the difference between any two numbers is known. For inſtance, the operation whereby the difference between 7842 and 6458 is known to be 1384, is ſubtraction.

The greater number is the minuend, the leſſer the ſubtrahend, and the difference the remainder.

Having placed units under units, &c. take every figure in the ſubtrahend from the figure over it in the minuend.

EXAMPLES.

Minuend	6847638
Subtrahend	4646216
Remainder	2201422

From	96786386
Take	62472234

When any figure in the minuend is leſſer than that under it, add 10 to it, and call the next figure to the left 1 leſs; or, which has the ſame effect, call the next figure in the ſubtrahend one more.

For 10 in any denomination is equal to 1 in the next.

EXAMPLES.

Minuend	7624674	Minuend	6297368
Subtrahend	4272413	Subtrahend	4734926
Remainder	3352261	Remainder	1562442

In the fifth place of the firſt example, 7 is under 2, therefore add 10 to 2, and take 7 from the ſum. Take 1 from the next 6 for the 10 which was added to the 2; becauſe 10 in the fifth place is equal to 1 in the ſixth. Or add 1 to the next 2, for 1 added to the ſubtrahend has the ſame effect as 1 taken from the minuend.

From	7432846	From	8450764
Take	4786234	Take	7930486

From	8070346	From	2000000
Take	6429524	Take	1750394

From	94320009	From	1000000
Take	672407	Take	777777
Remainder	93647602		

A piece of money being coined in 1734, how old is it in 1779?

1779
1734

Anſwer 45 *years.*

His Majeſty King George the Third was born in 1738. How old will he be in 1780?

Anſwer 42 *years.*

A leaſe was drawn in 1759, to continue 99 years. —How long is it ſince it was drawn, when will it expire, and how long to remain in force, this being 1779?

Anſwer, 20 years ſince it was drawn. It will expire in 1858, to which time it is 79 years.

An army at the beginning of a campaign conſiſted of 73200 men. During the ſummer three reinforcements of 6450 men each arrived. Seven thouſand men were killed, 671 taken by the enemy, 73 deſerted, and the ſick were 1718. How many effective men remained at the end of the campaign?

Solution. Multiply 6450 by 3, and add the product to 73200. From the ſum take the amount of the killed, priſoners, deſerters, and ſick, and the remainder 82084 is the anſwer.

SUBTRACTION
OF
MONEY.

TAKE the farthings from the farthings, the pence from the pence, &c. and when there are more farthings in the ſubtrahend than in the minuend, add 4 to the minuend, which is borrowing 1 from the pence.

To the pence in the minuend, when neceſſary, add 12; and 20 to the ſhillings in the minuend, when there is occaſion. The pounds are ſubtracted like other whole numbers.

EXAMPLES.

	l.	s.	d.
Minuend ……	762	17	8¾
Subtrahend ..	642	10	5½
Remainder £	120	7	3¼

From ……….	3647	14	8¼
Take ……….	2742	16	4¾
Remainder £	904	18	3½

From ……….	1921	17	9½
Take ……….	1708	0	10¾

	l.	s.	d.
Minuend ….	968	11	7½
Subtrahend ..	672	8	2¼

From ……….	6280	11	3½
Take ……….	4627	18	7¼

From ……….	9647	11	6
Take ……….	8738	17	8

	l.	*s.*	*d.*
From	2000	0	0
Take	972	14	9¾

	l.	*s.*	*d.*
From	7367	12	9¾
Take	6000	0	0

Subtract 3716*l.* 17*s.* 6¾*d.* from 4027*l.* 10*s.* 9*d.*

	l.	*s.*	*d.*
	4027	10	9
	3716	17	6¾
Remainder £	310	13	2¼

The three following queſtions require both addition and ſubtraction.

A gentleman paid, for rum 16*l.* 12*s.* 8*d.* for wine 11*l.* 19*s.* 6*d.* and for brandy fourteen guineas. How much remained from 50*l.*?

	l.	*s.*	*d.*
Rum	16	12	8
Wine	11	19	6
Brandy	14	14	0
Sum .... £	43	6	2

	l.	*s.*	*d.*
Caſh at firſt ..	50	0	0
Total paid	43	6	2
Anſwer £	6	13	10

A houſekeeper paid for green peaſe 4*s.* 7½*d.* cauliflowers 2*s.* 6*d.* gooſeberries 1*s.* 6*d.* veal 9*s.* 9*d.* fowls 7*s.* 6*d.* and eggs 2*s.* 3½*d.* How much remained from two guineas?

Anſwer 13*s.* 10*d.*

A merchant bought three parcels of goods for 1000*l.* —He ſold the firſt for 278*l.* 10*s.* 6*d.* the ſecond for 472*l.* 17*s.* ~~3*d.* and~~ the third for four hundred and three pounds. How much ~~did he~~ gain?

Anſwer 154*l.* 7*s.* 9*d.*

SUB-

SUBTRACTION
OF
WEIGHTS and MEASURES.

WEIGHTS and meaſures are ſubtracted in the ſame manner as money, only regard muſt be paid to their reſpective tables.

Troy Weight.

	lb.	oz.	dwt.	gr.
From	724	8	11	16
Take	647	6	10	22

Land Meaſure.

	a.	r.	p.
From	7832	3	12
Take	6073	2	16

Dry Meaſure.

	qrs.	b.	g.
From	84	5	6
Subtract	76	6	7

Avoirdupoiſe Weight.

	t.	c.	qr.	lb.	oz.	dr.
From ..	52	12	2	10	8	0
Take ..	26	16	2	16	2	14
	25	15	3	22	5	2

Long Meaſure.

	m.	f.	p.
From	276	2	19
Take	194	6	12

Time.

	h.	m.	s.
From	23	16	18
Take	16	18	13

To prove ſubtraction, add the remainder to the ſubtrahend, the ſum will be equal to the minuend, if right.—Or ſubtract the remainder from the minuend, the difference will be equal to the ſubtrahend.

DIVISION.

BY diviſion is known how often one number is contained in another. For inſtance, the operation whereby it is known that 54 is contained 32 times in 1728, is diviſion.

The number given to be divided, is called the dividend; the number whereby it is divided, is called the diviſor; and the number of times which the diviſor is contained in the dividend, is the quotient.

If the diviſor is leſſer than the ſame number of figures in the former part of the dividend, that period of the dividend is the leading period; thus 745 is the leading period in the firſt example.

But if the diviſor is greater than the ſame number of figures in the dividend, then one figure more of the dividend is required to conſtitute the leading period; for inſtance, 4053 is the leading period of the ſecond example.

There are ſeveral rules for dividing numbers: the following is the moſt regular one.

Determine how often the diviſor is contained in the leading period; place the figure, expreſſing the number of times, in the quotient, and multiply it into every figure of the diviſor, beginning at the units, and placing the firſt figure of the product under the laſt figure, on the right of the leading period; ſubtract, and to the remainder bring, from the dividend, the figure next to the leading period. The remainder, with this figure joined to it, is a minuend.

Find how often the diviſor is contained in the minuend, place the figure expreſſing the number of times in the quotient, and multiply it into the diviſor, ſetting the product under the minuend. Subtract, and to the remainder bring from the dividend the figure next to that already taken down, and a new minuend is thereby formed.

Enquire how often the diviſor is contained in this new minuend, place the figure in the quotient, multiply it into the diviſor, ſubtract the product from the new minuend, and to the remainder bring the next figure in the dividend.

Proceed in this manner until all the figures in the dividend are uſed.

EXAMPLE I.

Divide 7452 by 324.

OPERATION.

```
324 ) 7452 ( 23
      648
      ———
       972
       972
      ———
       ...
      ———
```

The leading period is 745, in which the diviſor is contained twice, therefore ſet 2 in the quotient, and multiply it into the diviſor, beginning at the 4. The whole product 648, taken from the leading period, leaves 97, to which the next figure of the dividend, 2, being joined, 972 is a minuend. In 972 the diviſor 324 is contained thrice, therefore 3 being ſet in the quotient, and multiplied into the diviſor, the product is 972, conſequently 324 is contained exactly 23 times in 7452.

When the product, or ſubtrahend, is greater than the leading period, or than the minuend, the quotient figure is too large, and muſt be altered to a leſſer.

But if any remainder, before a figure is taken from the dividend, be greater than the diviſor, the laſt quotient figure is too ſmall, and muſt be altered to a larger.

EXAMPLE II.

Divide 4053672 by 756.

OPERATION A.	OPERATION B.
756) 4053672 (5362	756) 4053672 (5000
3780	3780000
2736	273672 (300
2268	226800
4687	46872 (60
4536	45360
1512	1512 (2
1512	1512
....	....

The leading period is 4053, for 756 is greater than 405, and therefore one figure more is required to conſtitute the leading period.

The theory of diviſion is ſeen in the operation B. There are three figures in the dividend beſide the leading period, therefore the firſt figure 5 in the quotient is 5000. There are two figures in the remainder beſide the leading period of it, 2736; and conſequently the ſecond in the quotient, 3, is 300.

The diviſor is contained	5000	times in	3780000
	300		226800
	60		45360
	2		1512
Therefore	5362	times in	4053672

By comparing the two operations A and B, it is evident that they are eſſentially alike. For the ſake of conciſeneſs, the multiplication and ſubtraction of the cyphers are omitted in A, and only one figure taken down at once, being all that is neceſſary when the cyphers are neglected.

More EXAMPLES.

Divide 3909648 by 6932.
The quotient is 564.

Divide 47839375 by 76543.
The quotient is 625.

Divide 22497588 by 3726.
Quotient 6038.

When the diviſor is greater than the minuend, place a cypher in the quotient; bring another figure to the minuend from the dividend, and proceed as uſual. If there be a neceſſity for bringing two or more figures extraordinary from the dividend, place two or more cyphers in the quotient.

Divide 3714322846 by 74282.
Quotient 50003.

Divide 114488640 by 216.
Quotient 530040.

When, as in the two following examples, the laſt figures of the diviſor are cyphers, diſregard them, and neglect as many of the laſt figures of the dividend, which muſt be at laſt joined to the remainder; or if there be no other remainder, theſe figures make one.

Divide 753462 by 4200.
The quotient is 179, and the remainder is 1662.

Divide 9628462 by 83000.
The quotient is 116, and the remainder 462.

Divide 7324*l.* equally amongſt 32 men.
Anſwer—Each man's ſhare is 229*l.*

Divide 8784*l.* equally amongſt 108 men.

OPERATION.

```
           l.     l.  s.  d.
108 ) 8784 ( 81   6   8
      864
      ----
       144
       108
      ----
        36
         20
      ----
       720
       648
      ----
        72
        12
      ----
       864
       864
      ----
       ...
      ----
```

By dividing the pounds it ſoon appears that every man is to receive 81*l.* beſide his ſhare of the remaining 36*l.* which are multiplied by 20 to reduce them into ſhillings; theſe are divided by 108, by which it is known that every man's part of the 36*l.* is 6*s.* beſide the ſhare of the 72*s.* remainder, which are multiplied by 12 to reduce them into pence; theſe are divided by 108, and the quotient is 8*d.* conſequently the entire ſhare of each man is 81*l.* 6*s.* 8*d.*

Divide 37429*l.* equally amongſt 56 men.

Each man 668*l.* 7*s.* 6*d.*

Divide 2193*l.* 12*s.* 11¼*d.* equally amongſt 63 men.

Each man's ſhare 34*l.* 16*s.* 4¾*d.*

When the ſum to be divided contains ſhillings, *&c.* multiply the remaining pounds, as before, by 20, and at the ſame time add the given ſhillings, 12, to the product. When the remaining ſhillings are multiplied by 12, add at the ſame time the given pence, 11, to the product; and when the remaining pence are multiplied by 4, add the given farthing, 1.

Divide 5200*l.* 3*s.* 7½*d.* equally amongſt 99 men.

The ſhare of every man is 52*l.* 10*s.* 6½*d.*

Divide 994*l.* 19*s.* 11¼*d.* equally amongſt 52 men.

Every man's ſhare is 19*l.* 2*s.* 8¼*d.* and 9 farthings remain.

SHORT DIVISION.

If the diviſor is leſs than 13, there is no occaſion to ſet down the operation, but only the reſult, or quotient.

Divide 7452 by 3.

```
3 ) 7452
    ————
    2484 Quotient.
```

The diviſor 3 is contained twice in 7, and 1 remains; ſet down 2, and ſuppoſe the remainder 1, to ſtand before 4; in 14, the diviſor 3 is contained 4 times, and the remainder is 2; in 25, 3 is contained 8 times, and 1 remains; and in 12, 3 is contained 4 times.

Divide 658431 by 9.

```
9 ) 658431
    ——————
     73159 Quotient.
```

Divide 678642 by 12.

Anſwer, the quotient is 56553, and 6 remains.

Divide 8636842 by 8000.

Diſregard the cyphers, as in *page* 40, and neglect the three laſt figures of the dividend.—The quotient is 1079, and remainder 4842.

Divide 6428*l.* equally amongſt three ſiſters.

The ſhare of each, is 2142*l.* 13*s.* 4*d.*

Diviſion in general is proved by multiplying the quotient by the diviſor. If the product be equal to the dividend, the operation is right.

ARITHMETIC.

BOOK II.

Reduction, Vulgar Fractions, and Decimal Fractions.

REDUCTION.

BY reduction, money, weights, and meaſures, are changed from one denomination to another, and it is either deſcending or aſcending.

REDUCTION DESCENDING.

When the alteration is from a higher denomination to a lower, as from pounds to ſhillings, the reduction is deſcending, and is performed by multiplying the given ſum, weight, or meaſure, by the denominative parts of the integer, or by ſuch of theſe parts as appear by the nature of the queſtion to be neceſſary.

EXAMPLES.

EXAMPLES.

How many farthings are in 7484*l.* ?

In this example, one pound is the integer, whoſe denominative parts are 20, 12, and 4.

```
            l.
          7484
            20  ſhillings in 1 pound.
       -------
        149680  ſhillings in 7484 pounds.
            12  pence in 1 ſhilling.
       -------
       1796160  pence in 7484 pounds.
             4  farthings in 1 penny.
       -------
Anſwer 7184640  farthings in 7484 pounds.
       -------
```

How many farthings are in 424*l.* ?

$424 \times 20 \times 12 \times 4 = 407040$ farthings.

The meaning of the above expreſſion is, that 424*l.* is multiplied by 20, the product by 12, and this product by 4.

How many farthings are in 3000*l.* ?

Anſwer 2880000.

How many pence are in 512*l.* ?

The multipliers to be uſed in this example, are 20 and 12, as the reduction only deſcends to pence.

$512 \times 20 \times 12 = 122880$ pence.

How many pence are in 7327*l.*?

7327 × 20 × 12 = 1758480 pence.

In 819*l.* how many ſhillings?

819 × 20 = 16380 ſhillings.

In 7082*l.* how many ſhillings?

Anſwer 141640 ſhillings.

In 4325 ſhillings, how many farthings?

The integer in this example is one ſhilling, whoſe denominative parts are 12 and 4.

4325 × 12 × 4 = 207600 farthings.

In 4440 ſhillings, how many farthings?

Anſwer 213120 farthings.

In 322 ſhillings, how many pence?

332 × 12 = 3984 pence.

How many farthings are in 327*l.* 17*s.* 8½*d.*?

OPERATION.

l.	*s.*	*d.*
326	17	8½
20		
6537		
12		
78452		
4		
313810 farthings.		

At the ſame time that the pounds are multiplied by 20, add the 17*s.* thus,—Inſtead of the cypher ſet down 7, and add 1 to the product of 2 by 6. To the product of 12 times 7, add the 8*d.* and to the product of 4 times 2, add the 2 farthings.

How many farthings are in 8452*l.* 18*s.* 3½*d.*?
Anſwer 8114798 farthings.

How many ſhillings are in 728*l.* 13*s.*?
Anſwer 14573 ſhillings.

How many ſhillings are in 1070*l.* 8*s.*?

How many pence are in 902*l.* 14*s.* 3*d.*?
Anſwer 216651 pence.

In 3200*l.* 12*s.* 3*d.* how many pence?

In 17*s.* 5½*d.* how many farthings?
Anſwer 838 farthings.

In 13*s.* 2*d.* how many pence?
Anſwer 158 pence.

In 19*s.* 7*d.* how many pence?

In 249 guineas, how many farthings?

The integer is one guinea, whoſe denominative parts are 21, 12, and 4.

249 × 21 × 12 × 4 = 250992 farthings.

In 324*lb.* troy, how many grains?

The integer is 1*lb.* troy, whoſe d. parts are 12, 20, and 24.

324 × 12 × 20 × 24 = 1866240 grains.

In 372*lb.* 8*oz.* 16*dwt.* 14*gr.* how many grains?

lb. oz. dwt. gr.
372 8 16 14
12

4472 ounces.
20

89456 pennyweights.
24

357838
178912

Anſwer 2146958 grains.

To the product of 12 times 2, add the 8 *oz.* At the time of multiplying the ounces by 20, add the 16 *dwt.* in the ſame manner that ſhillings are added in *page* 45. When the pennyweights are reduced into grains, to the product of 4 times 6, add the 14 grains.

In 403*lb.* 5*oz.* 13*dwt.* 18*gr.* how many grains?
Anſwer 2324010 grains.

How many drams are in 364 tons avoirdupoiſe?

The integer is 1 ton, the d. parts are 20, 4, 28, 16, 16. *See the third table*, page 7.

$364 \times 20 \times 4 \times 28 \times 16 \times 16 = 208732160$ drams.

In 32*t.* 13*c.* 2*qr.* 19*lb.* 13*oz.* 12*dr.* how many drams?
Anſwer 18742236 drams.

The 13*c.* 2*qr.* *&c.* are added in the ſame manner as the ounces, pennyweights, *&c.* in troy weight.

In 19*c.* 3*qr.* 18*lb.* how many pounds?
Anſwer 2230 pounds.

One hundred weight is the integer, the d. parts of which, neceſſary in this example, are 4 and 28.

How many pounds are in 31*c.* 2*qr.* 19*lb.*?
Anſwer 3547 pounds.

How many drams are in 27 pounds?
27 × 16 × 16 = 6912 drams.

Sixteen is a number which often occurs in buſineſs, either as a multiplier, or a diviſor;—the operation where it does occur will be greatly ſhortened if the ſcholar has the following table off.

1	2	3	4	5	6	7	8	9
16	32	48	64	80	96	112	128	144

How many poles or perches are in 1735 acres?
1735 × 4 × 40 = 277600 poles.

How many poles are in 455*a.* 3*r.* 14*p.*?
Anſwer 72934 poles.

Reduce 42 miles into inches?

42 × 8 × 40 × $\frac{11}{2}$ × 3 × 12 = 2661120 inches.——*See the third table*, page 8.

As 11 is exactly twice $5\frac{1}{2}$, any number multiplied by 11, will produce twice as much as if multiplied by $5\frac{1}{2}$; therefore, if the poles be multiplied by 11, and the product divided by 2, the yards will be found.

Required the inches in 36*m*. 7*f*. 26*p*. 4*yd*. 2*f*. 10*i*.?

Anſwer 2341726 inches.

The 4 yards muſt be added after the diviſion by 2.

Required the pints in 13 tuns of wine?

13 × 4 × 63 × 8 = 26208 pints.

REDUCTION ASCENDING.

WHEN the alteration is from a lower denomination to a higher, as from farthings to pounds, the reduction is aſcending, and is performed by dividing the given ſum, weight, or meaſure, by the denominative parts of the integer, or by ſuch of theſe parts as appear, by the nature of the queſtion, to be neceſſary.

EXAMPLES.

In 7184640 farthings, how many pounds?

In this example, one pound is the integer; the d. parts are 20, 12, and 4, which are therefore the ſeveral diviſors, beginning with 4; and are ſuch, that the operation may be performed by ſhort diviſion.

4) 7184640 farthings.

12) 1796160 pence.

20) 149680 ſhillings.

7484 pounds.

The foregoing example is the converſe of the firſt example of reduction deſcending, and is a proof of it. In general, the one reduction may be proved by the other.

How many pounds are in 407040 farthings?
Anſwer 424 pounds.

How many pounds are in 635520 farthings?
Anſwer 662 pounds.

How many pounds are in 736499 farthings?

4) 736499 $\frac{3}{4}$ remain.
12) 184124 8*d.* remain.
2,0) 1534,3 3*s.* remain.
Anſwer 767*l.* 3*s.* 8$\frac{3}{4}$*d.*

How many pounds are in 826402 farthings?
Anſwer 860*l.* 16*s.* 8$\frac{1}{2}$*d.*

How many pounds are in 122880 pence?
The diviſors for ſolving this example, are 12 and 20.
Anſwer 512 pounds.

How many pounds are in 97328 pence?
Anſwer 405*l.* 10*s.* 8*d.*

In 79239 ſhillings, how many pounds?
No diviſor but 20 is here neceſſary.
Anſwer 3961*l.* 19*s.*

How many guineas are in 250992 farthings?

One guinea being the integer, the diviſors are 4, 12, and 21, the laſt of which will require long diviſion.

Anſwer 249 guineas.

How many guineas are in 60000 pence?
Anſwer 238*g*. 2*s*.

How many crowns are in 73452 pence?
Anſwer 1224*c*. 1*s*.

In 18742236 drams avoirdupoiſe, how many tuns?

The diviſors are 16, 16, 28, 4, and 20.

16) 18742236 12*dr*. remain.
16) 1171389 13*oz*. remain.
28) 73211 19*lb*.
4) 2614 2*qr*.
20) 653 13*c*.

Anſwer 32*t*. 13*c*. 2*qr*. 19*lb*. 13*oz*. 12*dr*.

In 34728692 drams, how many tuns?
Anſwer 60*t*. 11*c*. 0*qr*. 26*lb*. 15*oz*. 4*dr*.

In 2230 pounds, how many hundred weight?
Anſwer 19*c*. 3*qr*. 18*lb*.

How many hundred weight are in 27402 pounds?

Anſwer 244*c.* 2*qr.* 18*lb.*

How many pounds are in 30024 drams?

Anſwer 117*lb.* 4*oz.* 8*dr.*

In 64324 poles, how many acres?

Anſwer 402*a.* or. 4*p.*

How many weeks are there from March 9th, to Auguſt 26th, both incluſive?

Remain in March, ...	23 days.
April, ...	30
May, ...	31
June, ...	30
July, ...	31
Auguſt, ...	26

7) 171 days.

Anſwer 24*w.* 3*d.*

VULGAR

VULGAR FRACTIONS.

A VULGAR FRACTION is denoted or expreſſed by a numerator and a denominator.

The denominator ſhews the number of parts into which unity or one is conceived to be divided. The numerator ſhews the number of theſe parts expreſſed by the fraction. In other words, the numerator may be conſidered as a dividend, and the denominator as the diviſor.

Thus $\frac{3}{4}$ of a pound is the fractional expreſſion for three times the one-fourth part of a pound. Or it is an expreſſion for 3*l.* divided by 4.

Again, $\frac{7}{8}$ ſtand for ſeven-eighths, or ſeven times the one-eighth of any integer; or, which is the ſame, it is an expreſſion for 7 divided by 8.

The above two fractions are proper ones, becauſe the numerator of each is leſſer than its denominator.

When the numerator is greater than the denominator, it is an improper fraction, as $\frac{9}{3}$.

A compound fraction is a part of a part, as $\frac{4}{5}$ of $\frac{2}{9}$.

A mixt number contains both a whole number and a fraction, as $32\frac{7}{14}$.

The value of a fraction conſiſts in the proportion between the numerator and denominator: if, for inſtance, the denominator be thrice as much as the numerator,

merator, the fraction is one-third; for one-third of the parts of the denominator are then expreſſed by the numerator. Hence there may be an infinity of fractional expreſſions, all of the ſame value; thus $\frac{2716}{8148}$, $\frac{752}{2256}$, $\frac{174}{522}$, $\frac{1}{3}$, *&c.* are all equal in value, the denominator of each being thrice its numerator.

REDUCTION

OF

VULGAR FRACTIONS.

REDUCTION is the firſt part of fractions, becauſe it prepares them for addition, ſubtraction, *&c.* It contains ſeven caſes.

CASE I.

To abbreviate fractions, that is, to expreſs them in their leaſt terms, and to retain their primitive value.

Divide both numerator and denominator by their greateſt common diviſor, *viz.* the greateſt diviſor which can divide each of them, without a remainder.

That the new fraction is equal to the given one, is plain, for the proportion between the numerator and denominator is not altered when both are divided alike.

EXAMPLES.

EXAMPLES.

Reduce $\frac{32}{40}$ into its leaſt terms.

Anſwer $\frac{4}{5}$, for 8 is the common diviſor.

Reduce $\frac{60}{72}$ into its leaſt terms.

Anſwer $\frac{5}{6}$, for 12 is the common diviſor.

Reduce $\frac{45}{63}$ into its leaſt terms.

Anſwer $\frac{5}{7}$, for 9 is the common diviſor.

Reduce $\frac{1320}{1920}$ into its leaſt terms.

Anſwer $\frac{11}{16}$, for 120 is the common diviſor; or, reject a cypher from each, and 12 is the diviſor.

Reduce $\frac{3200}{24000}$ into its leaſt terms.

Anſwer $\frac{2}{15}$, for 1600 is the common diviſor; or, reject two cyphers from both parts of the fraction, and 16 is the diviſor.

When the greateſt common diviſor does not immediately appear, divide by any diviſor, and the new fraction by any diviſor; and proceed until a fraction is found, whoſe parts cannot be farther divided by a common diviſor.

Reduce $\frac{312}{336}$ into its leaſt terms.

Divide both parts by 2, and the new fraction will be $\frac{156}{168}$; divide again by 2, and the reſult will be $\frac{78}{84}$; each part whereof is diviſible by 6, and the leaſt terms is $\frac{13}{14}$.

Reduce $\frac{512}{2304}$ into its leaſt terms.

Anſwer $\frac{2}{9}$.

If the denominator be diviſible by the numerator, the numerator of the leaſt terms is 1, and the denominator is the quotient of the diviſion.

Reduce $\frac{328}{3936}$ into its leaſt terms.
Anſwer $\frac{1}{12}$, for both parts are diviſible by 328.

Reduce $\frac{32}{128}$ into its leaſt terms.
Anſwer $\frac{1}{4}$.

When the units place of both numerator and denominator is 5, or when it is 5 in one of them, and a cypher in the other, both parts may be divided by 5.

Reduce $\frac{345}{405}$ into its leaſt terms.
Anſwer $\frac{23}{27}$, for 5 is a firſt diviſor, and 3 a ſecond one.

Reduce $\frac{2205}{2520}$ into its leaſt terms.
Anſwer $\frac{7}{8}$.

CASE II.

To find the value of a proper fraction, in coin, weight, or meaſure.

Multiply the numerator by the greateſt denominative part of the integer, and divide by the denominator. Multiply the remainder, if any, by the next lower denominative part, and divide by the denominator. Proceed in the ſame manner until either there is no remainder, or the denominative parts are all uſed.

For the numerator cannot be actually divided by the denominator, until reduced into a lower denomination.

EXAMPLES.

EXAMPLES.

Required the value of $\frac{13}{16}$ of a pound. That is, if a pound be divided into 16 equal parts, the value of 13 of theſe parts is required?

OPERATION.

```
     13  numerator.
     20  greateſt denominative part of a pound.
    ----  s.  d.
16) 260 (16  3 Anſwer.
    16
    ---
    100
     96
    ---
    ..4
     12
    ---
16)  48 (3
     48
    ---
     ..
    ---
```

Required the value of $\frac{7}{32}$ of a pound?

Anſwer 4*s.* $4\frac{1}{2}$*d.*

How much is $\frac{2}{3}$ of a pound?

Anſwer 13*s.* 4*d.*

How much is $\frac{7}{8}$ of a pound?

Anſwer 17*s.* 6*d.*

When the integer properly reduced, can be readily divided by the denominator, it is generally beſt to do ſo, and multiply the quotient by the numerator.

For inſtance, in the preceding example, 1*l.* divided 8, is 2*s.* 6*d.* which multiplied by 7, produces 17*s.* 6*d.* the ſame as by the general method.

Required the value of $\frac{9}{10}$ of a pound?

Anſwer 18*s.*

Required the value of $\frac{25}{72}$ of a pound?

Anſwer 6*s.* 11*d.* 1*qr.* $\frac{1}{3}$

Required the value of $\frac{75}{504}$ of a guinea?

Anſwer 3*s.* 1$\frac{1}{2}$*d.*

How much is the value of $\frac{5}{63}$ of a guinea?

Anſwer 1*s.* 8*d.*

Required the quarters, pounds, &*c.* in $\frac{1361}{3584}$ of a hundred weight?

Anſwer 1*qr.* 14*lb.* 8*oz.* 8*dr.*

Required the value of $\frac{35}{224}$ of a hundred weight?

When a fraction can be reduced into lower terms, it is generally beſt to reduce it.

$\frac{35}{224} = \frac{5}{32} =$ 17*lb.* 8*oz.*

How many roods and perches are in $\frac{13}{36}$ of an acre?

Anſwer 1*r.* 17*p.* $\frac{7}{9}$.

CASE

CASE III.

To find the value of an improper fraction.

Divide the numerator by the denominator, the quotient is a whole number of the ſame denomination with the integer. Find the value of the remainder, if any, by the preceding caſe.

For if the numerator is equal to the denominator, the fraction is equal to 1, becauſe all the parts into which unity is conceived to be divided, are expreſſed by the fraction. If the numerator be twice as much as the denominator, the fraction is equal to 2, becauſe twice the number of parts into which unity is conceived to be divided, are expreſſed by the numerator. And ſo on.

EXAMPLES.

Required the value of $\frac{36}{4}$ of a pound?
Anſwer 9 pounds.

Required the value of $\frac{98}{15}$ of a pound?
Anſwer 6*l.* $\frac{8}{15}$ = 6*l.* 10*s.* 8*d.*

Turn $\frac{75}{16}$ into a mixt number.
Anſwer $4\frac{11}{16}$.

CASE IV.

To turn a mixt number into an improper fraction.

Multiply the whole number by the denominator; add the numerator, and ſubſcribe the denominator fractionwiſe under the ſum; for this caſe is the converſe of the preceding one.

EXAMPLES.

Reduce 9 $\frac{4}{5}$ into an improper fraction.

Anſwer $\frac{49}{5}$.

Reduce 65 $\frac{7}{9}$ into an improper fraction.

Anſwer $\frac{592}{9}$.

Reduce 43 $\frac{24}{35}$ into an improper fraction.

Anſwer $\frac{1529}{35}$.

CASE V.

To turn any coin, weight, or meaſure, into the part or fraction of a given integer.

Reduce the coin, *&c.* into the leaſt given denomination, and reduce the integer into the ſame denomination. Place the former for the numerator, and the latter for the denominator of the required fraction.

For the denominator ſhews the parts into which the integer is divided, and the numerator expreſſes the number of theſe parts contained in the required fraction.

EXAMPLES.

Reduce 13*s.* 4*d.* into the part or fraction of a pound.

In 13*s.* 4*d.* are 160 pence, and in 1 pound there are 240 pence, therefore the fraction is $\frac{160}{240}$ of a pound; the leaſt terms is $\frac{2}{3}$, therefore 13*s.* 4*d.* is two-thirds of a pound; that is, if a pound be divided into three equal parts, 13*s.* 4*d.* is two of theſe parts.

Reduce 12*s.* 6*d.* into the fraction of a pound.

Anſwer $\frac{5}{8}$.

Reduce 3*s.* 11$\frac{1}{4}$*d.* into the fraction of a guinea.

In 3*s.* 11$\frac{1}{4}$*d.* are 189 farthings, and 1008 in a guinea, therefore $\frac{189}{1008}$ is the fraction, whoſe leaſt terms is $\frac{3}{16}$.

Reduce 6*oz.* 8*dr.* into the fraction of a pound.

Anſwer $\frac{13}{32}$.

Reduce 2*r.* 25*p.* into the fraction of an acre.

Anſwer $\frac{21}{32}$.

CASE VI.

To reduce a fraction of one denomination, to the fraction of another.

When the required denomination is the greater, multiply the proportion between the two integers into the denominator of the fraction. But if the required denomination is the leſſer, multiply into the numerator.

For it is eaſy to conceive, that the fraction of a pound muſt have a denominator 20 times as large as the denominator of the fraction of a ſhilling, the numerators of both being the ſame, and their values equal. The ſame may readily be conceived of other integers.

Reduce $\frac{3}{4}$ of a ſhilling into the fraction of a pound.

Anſwer $\frac{3}{80}$ of a pound is equal to $\frac{3}{4}$ of a ſhilling.

Reduce $\frac{5}{8}$ of a ſhilling into the fraction of a pound.

Anſwer $\frac{5}{160}$.

Turn $\frac{3}{4}$ of a penny into the fraction of a pound.

The proportion between the integers is 240, therefore the required fraction is $\frac{3}{960} = \frac{1}{320}$.

Turn $\frac{3}{80}$ of a pound into the fraction of a ſhilling.

Anſwer $\frac{60}{80} = \frac{3}{4}$, for $3 \times 20 = 60$.

Reduce $\frac{5}{120}$ of a pound into the fraction of a ſhilling.

Anſwer $\frac{100}{120} = \frac{5}{6}$.

Reduce $\frac{3}{16}$ of a guinea into the fraction of a pound.

Anſwer $\frac{63}{320}$ of a pound; for, multiply the numerator by 21, and it is thereby reduced into the fraction of a ſhilling, *viz.* $\frac{63}{16}$ of a ſhilling; which, by multiplying the denominator by 20, is reduced into the fraction of a pound, that is, $\frac{63}{320}$ of a pound.

Reduce $\frac{3}{4}$ of 6*d.* into the fraction of 9*d.*

Anſwer $\frac{1}{2}$ of 9*d.*

CASE VII.

To find a common denominator, that is, to find new fractions with a common denominator, which ſhall be reſpectively equal to given fractions of different denominations.

The product of all the denominators is the common denominator.

Multiply each numerator into all the denominators but its own, and the reſpective products are the new numerators, and muſt be ſeparately placed over the common denominator.

As every numerator is multiplied by the ſame numbers as its denominator, *viz.* by all the other denominators,

nators, it is very plain that the new fractions are reſpectively equal to the given ones; for when both numerator and denominator are multiplied by the ſame multiplier or multipliers, the value of the fraction is not altered. *See* CASE I.

EXAMPLES.

Reduce $\frac{2}{3}$, $\frac{4}{9}$, and $\frac{5}{7}$, into a common denomination. That is, find three fractions reſpectively equal to the above, and to have all their denominators the ſame with each other.

$3 \times 9 \times 7 = 189$, common denominator.
$2 \times 9 \times 7 = 126$, the firſt new numerator.
$3 \times 4 \times 7 = 84$, the ſecond new numerator.
$3 \times 9 \times 5 = 135$, third new numerator.

Therefore $\frac{126}{189} = \frac{2}{3}$.
$\frac{84}{189} = \frac{4}{9}$.
And $\frac{135}{189} = \frac{5}{7}$.

Reduce $\frac{4}{7}$, $\frac{8}{9}$, and $\frac{11}{13}$, into a common denomination.

Anſwer $\frac{416}{728} = \frac{4}{7}$.
$\frac{819}{728} = \frac{8}{9}$.
$\frac{616}{728} = \frac{11}{13}$.

When there are only two fractions, and their denominators diviſible by each other, multiply the proportion into both parts of the fraction whoſe denominator is the leſſer.

Reduce $\frac{7}{12}$ and $\frac{11}{36}$ into a common denomination.

The proportion between the denominators, found by dividing them, is 3, which multiplied into both parts

of

of the firſt fraction, it becomes $\frac{21}{36}$, whoſe denominator is the ſame as that of the other fraction.

Required a fraction equal to $\frac{3}{4}$, whoſe denominator ſhall be 48?

The proportion between the denominators is 12, which multiplied into 3, the given numerator, the product 36 is the required numerator, and therefore $\frac{36}{48}$ is the required fraction equal to $\frac{3}{4}$.

Or, multiply 48 by 3, and divide by 4, as by the nature of diviſion, it is the ſame whether a dividend is increaſed in a given proportion, or the quotient is increaſed in the ſame proportion. If the dividend be multiplied by 5, for inſtance, the quotient will be 5 times as much on that account.

Required a fraction equal to $\frac{7}{8}$, whoſe denominator ſhall be 1000?

To multiply 7 by 1000 is only to join three cyphers to it, conſequently $\frac{7 \times 1000}{8} = \frac{7000}{8} = 875$, and therefore $\frac{875}{1000}$ is the required fraction.

Required a fraction equal to $\frac{3}{16}$, to have an unit and cyphers for its denominator.

$$\begin{array}{r|l} 16 & 30000 \\ \hline & 1875 \end{array}$$

It requires four cyphers, that there may be no remainder; hence the denominator is 10000, and the numerator 1875, that is, the required fraction is $\frac{1875}{10000}$.

The theory of changing a vulgar fraction into a decimal one, is contained in the two preceding examples. *See* DECIMALS, *Caſe* II.

Reduce 31 $\frac{2}{3}$ and 8 $\frac{5}{8}$ into a common denomination.

Theſe reduced into improper fractions are $\frac{95}{3}$ and $\frac{69}{8}$, which, in a common denomination, are $\frac{760}{24}$ and $\frac{207}{24}$.

* * *

ADDITION

OF

VULGAR FRACTIONS.

WHEN the denominators are common, add all the numerators together, and divide by the common denominator.

For the ſum of all the numerators, placed fractionwiſe over the denominator, is a ſingle fraction, whoſe value is found as above. *See* CASES II. *and* III.

Add $\frac{4}{15}$, $\frac{9}{15}$, and $\frac{14}{15}$ of a pound, into one ſum.

The ſum of the numerators is 27, therefore the amount of the three fractions is $\frac{27}{15}$, whoſe value is readily found to be 1*l*, 16*s*.

Add $\frac{6}{32}$, $\frac{9}{32}$, $\frac{24}{32}$, and $\frac{10}{32}$ of a pound, into one ſum.

Anſwer 1*l.* 10*s.* $7\frac{1}{2}$*d.*

Add $\frac{3}{11}$, $\frac{7}{11}$, $\frac{12}{11}$, $\frac{9}{11}$, and $\frac{5}{11}$ into one ſum.

Anſwer $\frac{36}{11} = 3\,\frac{3}{11}$.

When the denominators are not common, reduce them to a common denomination, and divide the ſum of the new numerators by the common denominator.

Add $\frac{4}{5}$, $\frac{2}{3}$, and $\frac{7}{12}$ of a pound together.

The three fractions with a common denominator, reſpectively equal to the given ones, are found by CASE VII. to be $\frac{144}{180}$, $\frac{120}{180}$, and $\frac{105}{180}$. The ſum is $\frac{369}{180}$ = 2*l.* 1*s.*

Required the ſum of $\frac{11}{13}$, $\frac{17}{12}$, and $\frac{3}{5}$?

Anſwer $\frac{2233}{780} = 2\,\frac{673}{780}$.

Required the ſum of $51\,\frac{2}{5}$, $7\,\frac{4}{5}$, and $22\,\frac{1}{5}$?

The ſum of the fractions is $1\,\frac{2}{5}$, which being added to the whole numbers, the whole ſum is $81\,\frac{2}{5}$.

Add $\frac{7}{10}$, $\frac{2}{100}$, $\frac{3}{1000}$, and $\frac{5}{10000}$ into one ſum.

Each of theſe denominators being ten times as much as the preceding one, multiply both parts of the laſt fraction but one by 10; both parts of the laſt but two by 100, &c. and the given fractions become $\frac{7000}{10000}$, $\frac{200}{10000}$, $\frac{30}{10000}$, and $\frac{5}{10000}$, whoſe ſum is $\frac{7235}{10000}$; the numerator whereof is all the given numerators in a right line. This will evidently be always the caſe in theſe fractions, if the numerators be ſingle figures; for when the fractions are reduced to a common denomination, the firſt numerator will contain one cypher more than the ſecond, which will contain one cypher more than third, *&c.* conſequently, when added, the numerators will form a right line. To add ſuch fractions, therefore, nothing is neceſſary but to place the numerators in a right line, and for a denominator to place an unit and as many cyphers as the numerator conſiſts of figures.

Add $\frac{4}{10}$, $\frac{8}{100}$, $\frac{1}{1000}$, $\frac{2}{10000}$, and $\frac{5}{100000}$ of a pound, into one ſum.

Anſwer $\frac{48125}{100000}$ = 9*s.* 7½*d.*

The above fraction is readily valued, for the numerator being multiplied by 20, the product is divided by 100000, by cutting off five figures, *viz.* one for every cypher, or which is the ſame, one for every figure in the numerator; that on the left being the quotient, and thoſe on the right the remainder, to be multiplied by 12, *&c.*

OPERATION.

```
48125
   20
------
9,62500
     12
-------
7,50000
      4
-------
2,00000
-------
```

Add $\frac{3}{5}$, $\frac{3}{5}$, $\frac{3}{5}$, $\frac{3}{5}$, $\frac{3}{5}$, and $\frac{3}{5}$ together.

Anſwer $\frac{18}{5} = 3\frac{3}{5}$.

When the numerators are alike, their ſum is moſt readily found by multiplying one of them by their number. By the above example it is evident, that 6 times $\frac{3}{5}$ is $\frac{18}{5}$; that is, the product of a fraction by a whole number is found by multiplying the numerator by the whole number, and dividing by the denominator.

MULTIPLICATION OF VULGAR FRACTIONS.

I. TO multiply a fraction by a whole number.

Multiply the numerator by the whole number, and divide by the denominator. *See the preceding remark.*

EXAMPLES.

Multiply $\frac{9}{13}$ by 26.—The product is $\frac{234}{13} = 18$.

Multiply $\frac{7}{8}$ by 18.—The product is $\frac{126}{8} = 15\frac{6}{8} = 15\frac{3}{4}$.

Multiply $\frac{5}{9}$ by 9.—The product is 5. If, therefore, a fraction be multiplied by a number equal to its denominator, the product will be equal to its numerator.

II. To multiply a fraction by a fraction.

Multiply the numerators for the numerator of the product, and the denominators for the denominator of it.

EXAMPLES.

Multiply $\frac{3}{8}$ by $\frac{5}{6}$.—*Anſwer* $\frac{15}{48} = \frac{5}{16}$.

For 3 × 5 = 15, the numerator of the product.

And 8 × 6 = 48, the denominator of the product.

The product of $\frac{3}{8}$ by 5, by Case I. is $\frac{15}{8}$, but the product of $\frac{3}{8}$ by $\frac{5}{6}$, muſt only be one-ſixth of that product, and therefore $\frac{15}{8}$ muſt be divided by 6 times its preſent denominator, *viz.* 48. The ſame method of reaſoning is applicable to all other fractions, and therefore the above rule is a general one.

Multiply $\frac{21}{32}$ by $\frac{12}{42}$.—The product is $\frac{252}{1344} = \frac{3}{16}$.

Reduce $\frac{2}{3}$ of $\frac{7}{8}$, of $\frac{3}{5}$, into a ſingle fraction.
Anſwer $\frac{42}{120} = \frac{7}{20}$.

Multiply 6 $\frac{3}{5}$ by 8 $\frac{4}{9}$.

Theſe reduced into improper fractions are $\frac{33}{5}$ and $\frac{76}{9}$, whoſe product is $\frac{2508}{45} = 55\,\frac{33}{45} = 55\,\frac{11}{15}$.

Multiply 51 $\frac{2}{3}$ by $\frac{6}{7}$, and the product by 8.
Anſwer $\frac{7440}{21} = 354\,\frac{6}{21} = 354\,\frac{2}{7}$.

SUB-

SUBTRACTION
OF
VULGAR FRACTIONS.

WHEN the denominators are common, the difference between the numerators, divided by the denominator, is evidently the remainder.

EXAMPLES.

Take $\frac{2}{18}$ from $\frac{11}{18}$ of a pound.

```
     From 11
     Take  2
          ---
           9
          20
          ---
    18 ) 180 ( 10
          18
          ---
           ..0
          ---
```

Subtract $\frac{22}{32}$ from $\frac{29}{32}$.—The remainder is $\frac{7}{32}$.

If the denominators differ, reduce them into a common denomination.

Take $\frac{3}{5}$ from $\frac{11}{12}$.

The two fractions equal to thoſe given, are $\frac{36}{60}$ and $\frac{55}{60}$; conſequently the remainder is $\frac{19}{60}$.

Take $\frac{9}{11}$ from $\frac{12}{13}$.—The remainder is $\frac{108}{143}$.

When mixt numbers are to be ſubtracted, take the fractions from the fractions, and the whole numbers from the whole numbers; and, if neceſſary, borrow one from the whole numbers.

Subtract $92\frac{3}{16}$ from $100\frac{11}{16}$.——The remainder is $8\frac{8}{16} = 8\frac{1}{2}$.

Subtract $31\frac{12}{15}$ from $45\frac{9}{15}$.

In this example it is neceſſary to borrow 1, or, which is the ſame, $\frac{15}{15}$, this added to $\frac{9}{15}$, makes it $\frac{24}{15}$. The remainder, therefore, is $13\frac{12}{15} = 13\frac{4}{5}$.

In general, therefore, when the numerator of the ſubtrahend is the greater, add the denominator to the numerator of the minuend, ſubtract the numerator of the ſubtrahend from the ſum, place the denominator under the difference, and add 1 to the units of the ſubtrahend.

Subtract $9\frac{2}{3}$ from $21\frac{8}{15}$.——The remainder is $11\frac{39}{45} = 11\frac{13}{15}$.

DIVISION

OF

VULGAR FRACTIONS.

I. TO divide a fraction by a whole number.

Divide the numerator by the whole number, for the fraction will thereby be divided into as many parts as the whole number contains units.

EXAMPLES.

Divide $\frac{36}{11}$ by 9.—The quotient is $\frac{4}{11}$.

Divide $\frac{16}{25}$ by 4.—The quotient is $\frac{4}{25}$.

If the numerator cannot be divided by the whole number, multiply the denominator by the whole number; for increaſing the denominator has the ſame effect as decreaſing the numerator in the ſame proportion, the value of the fraction being equally diminiſhed in both caſes.

Divide $\frac{7}{8}$ by 5.—The quotient is $\frac{7}{40}$.

It is very obvious, that $\frac{7}{8}$ is 5 times as much as $\frac{7}{40}$, for the denominative parts of $\frac{7}{8}$ are 5 times as large as the denominative parts of $\frac{7}{40}$, and the numerators are equal.

Divide $\frac{12}{17}$ by 3.——The quotient is $\frac{12}{51} = \frac{4}{17}$.

II. To divide a fraction by a fraction.

Multiply the denominator of the diviſor by the numerator of the dividend, for the numerator of the quotient; and the numerator of the diviſor by the denominator of the dividend, for the denominator of the quotient.

Divide $\frac{16}{12}$ by $\frac{2}{3}$.——Quotient 2.

Diviſor.	Dividend.	Quotient.
$\frac{2}{3}$	$\frac{16}{12}$	$\frac{48}{24} = 2$.

If the dividend $\frac{16}{12}$ was divided by 2, the quotient would only be $\frac{16}{24}$ by the laſt CASE; but as it is divided by $\frac{2}{3}$, which is only one-third of 2, the quotient muſt be 3 times $\frac{16}{24}$, that is, $\frac{48}{24} = 2$, for the leſſer the diviſor, the greater the quotient.

Divide $\frac{6}{25}$ by $\frac{3}{5}$.—The quotient is $\frac{30}{75} = \frac{2}{5}$.

Divide $16\frac{4}{5}$ by $3\frac{2}{7}$.

The mixt numbers, when reduced to a common denomination, are $\frac{84}{5}$ and $\frac{23}{7}$, which being divided, the quotient is $\frac{588}{115} = 5\frac{13}{115}$.

Divide 6 by $\frac{5}{12}$.

The fractional expreſſion for 6, is $\frac{6}{1}$, conſequently the quotient is $\frac{72}{5} = 14\frac{2}{5}$.

DECIMALS.

A DECIMAL FRACTION has a dot or period placed before it, and has no denominator expreſſed, but is always underſtood to have one, conſiſting of an unit, and as many cyphers as the decimal contains figures.

Thus .7 ſtands for $\frac{7}{10}$. That is, if an unit, for inſtance, if one pound be divided into 10 equal parts, the decimal expreſſion for 7 of theſe parts, or 14*s.* is .7.

Alſo .64 ſtand for $\frac{64}{100}$, or $\frac{6}{10}$ and $\frac{4}{100}$.
For $\frac{6}{10} + \frac{4}{100} = \frac{64}{100}$. *See page* 66.

Likewiſe .865 is the decimal expreſſion for $\frac{865}{1000}$, or $\frac{8}{10}$, $\frac{6}{100}$, and $\frac{5}{1000}$; for $\frac{8}{10} + \frac{6}{100} + \frac{5}{1000} = \frac{865}{1000}$.

In general, the value of a decimal figure is ten times leſſer for every removal from the dot or period towards the right; thus, in the decimal .5555, &c. the 5 immediately after the dot is $\frac{5}{10}$, the next is $\frac{5}{100}$, the next is $\frac{5}{1000}$, &c.

REDUCTION

OF

DECIMALS.

REDUCTION of decimals contains three caſes.

CASE I.

To find the value of a decimal.

Multiply by the denominative parts of the integer, and cut off to the right at each multiplication as many figures as the given decimal contains. The figures on the left expreſs the value. *See page* 67.

EXAMPLES.

Required the value of .39375 of a pound?

```
 .39375
     20
 ------
7.87500
     12
-------
10.50000
       4
--------
 2.00000
--------
```

Anſwer 7*s*. 10½*d*.

The given decimal may be expreſſed $\frac{39375}{100000}$, hence the reaſon of cutting off five figures, for it is in effect to divide by 100000.

Required the value of .4875 of a pound?

Anſwer 9*s.* 9*d.*

Required the value of .75 of a pound?

Anſwer 15*s.*

Required the value of .00625 of a pound?

Anſwer 1½*d.*

How many pence and farthings are in .375 of a ſhilling?

Anſwer 4½*d.*

How many farthings are in .75 of a penny?

Anſwer ¾*qr.*

Required the value of .4375 of a guinea?

Anſwer 9*s.* 2¼*d.*

Required the value of .46875 of a tun, in hundred weights, quarters, *&c.*?

Anſwer 9*c.* 1*qr.* 14*lb.*

How many roods and poles are in .7354 of an acre?

Anſwer 2*r.* 37*p.* and .664 of a pole.

How many days are in .575 of a year?

Anſwer 209 days, and .875 of a day.

CASE II.

To turn or reduce a vulgar fraction to a decimal one.

Annex cyphers to the numerator, and divide by the denominator; for this will find the numerator of a fraction whoſe denominator is an unit and cyphers, *page* 64, that is, it will find a decimal; for the denominator of a decimal is underſtood to conſiſt of an unit and cyphers.

A decimal always conſiſts of as many figures as the denominator of it contains cyphers, therefore the number of the figures in the decimal muſt always be equal to the number of cyphers joined to the numerator; and when there is a deficiency, it muſt be ſupplied by prefixing cyphers.

EXAMPLES.

Reduce $\frac{15}{16}$ into a decimal.

Anſwer .9375.

Reduce $\frac{3}{25}$ into a decimal.

Anſwer .12.

Reduce $\frac{3}{64}$ into a decimal.

Anſwer .046875.

The decimal of $\frac{1}{4}$ is .25; the decimal of $\frac{1}{2}$ is .5; and .75 is the decimal of $\frac{3}{4}$.

Reduce $\frac{2}{3}$ into a decimal.

Anſwer .6666 +. This is called a repeating decimal, becauſe 6 may be repeated without end, which is evident by trying the diviſion.

Required the decimal of $\frac{2}{7}$?

Anſwer .285714285714, *&c.* This is called a circulating decimal, as 285714 will continually recur; for when theſe ſix figures are found, 2 remain, the ſame figure as the numerator.

CASE III.

To reduce any coin, weight, or meaſure, into a decimal.

Reduce it into a vulgar fraction by CASE V. *page* 60, and then into a decimal by the laſt CASE; that is, reduce the coin, *&c.* into the leaſt given denomination, add cyphers, and divide by the integer, reduced into the ſame denomination.

EXAMPLES.

Reduce 12*s.* 6*d.* into the decimal of a pound.
Anſwer .625, for $\frac{150}{240} = .625$.

Reduce 15*s.* 3*d.* into the decimal of a pound.
Anſwer .7625.

Reduce 13*s.* 1½*d.* into the decimal of a pound.
Anſwer .65625.

Reduce 1*s.* 12*s.* and 15*s.* into the decimals of a pound.

Anſwer, the decimal of 1*s.* is .05, for $\frac{1.00}{20} = .05$; the decimal of 12*s.* is .6, for $\frac{12.0}{20} = .6$; and the decimal of 15*s.* is .75.

The decimal of any even number of ſhillings is evidently expreſſed by half their number, thus .5 is the decimal of 10*s.* and the decimal of any odd number of ſhillings is half their number, with 5 joined to it; for inſtance, the decimal of 17*s.* is .85. A.

From the decimal of 1*s.* the decimal of any number of pence under 12, a pound the integer, may be readily derived. Divide .05 by 2, and the quotient .025 is the decimal of 6*d.* Divide .05 by 3, and the quotient .016666 is the decimal of 4*d.* Divide .05 by 4, 6, 8, and 12, and the quotients .0125, .008333, .00625, and .004166, are the decimals of 3*d.* 2*d.* 1½*d.* and 1*d.* reſpectively.

By addition, &c. the decimals of 11*d.* 10*d.* 9*d.* &c. are determined from the above. By a proper uſe of the following table, the decimal of any ſum may readily be found.

Pence.	Decimals, 1*l.* the integer.	Pence	Decimals, 1*l.* the integer.	Pence.	Decimals, 1*l.* the integer.
1	.00416 +	6	.025	11	.04583 +
2	.00833 +	7	.02916 +	Farthings.	
3	.0125	8	.03333 +	1	.0010416 +
4	.01666 +	9	.0375	2	.0020833 +
5	.02083 +	10	.04166 +	3	.003125

Required the decimal of 19*s.* 9¾*d.* by the table?

The decimal of 19*s.* is, by remark A ...... .95
——————— 9*d.* .... .0375
——————— ¾*qr.* .... .003125

Anſwer .990625

Where + is placed, the laſt figure may be repeated to any number of places; but there never can be any occaſion to carry the decimal farther than is done in the table.

Required the decimal of 7*s.* 10½*d.* one guinea the integer?

Neither this nor any of the following queſtions are done by the table.

In 7*s.* 10½*d.* are 378 farthings, and in one guinea 1008, therefore $\frac{378}{1008} = .375$.

Reduce 9*d.* into the decimal of a ſhilling.

Anſwer .75, for $\frac{9.00}{12} = .75$.

Reduce 8½*d.* into the decimal of a ſhilling.

Anſwer .7083 +.

Required the decimal of 2*qr.* 14*lb.* one hundred weight the integer?

Anſwer .625, for $\frac{70.000}{112} = .625$.

Required the decimal of 3 inches, one foot the integer?

Anſwer .25.

Reduce 2*r.* 28*p.* into the decimal of an acre.

Anſwer .675.

ADDITION
OF
DECIMALS.

PLACE units under units in the whole numbers, tenths under tenths in the decimals, ſo that the decimal dots may be right above each other, then proceed as in common addition; for 10 in any decimal line make one in the next, in decimals as well as in whole numbers.

EXAMPLES.

£	£
734.25	62.67
627.473	.742
72.7482	7.207
6.27	75.67
1440.7412	

Add 43.75, 629.4, 7.281, and 927.476 into one ſum.

Anſwer 1607.907.

Add 3333, 333.3, 33.33, 3.333, and .3333 into one ſum.

Anſwer 3703.2963.

l. *l.* *l.* *s.* *d.* *l.*
Add 76.25, 61.475, 42 12 6, and $52\frac{3}{4}$ into one ſum.

The decimal of 12*s.* 6*d.* is .625; the decimal of $\frac{3}{4}$ is .75; and the whole ſum 233*l.*1 = 233*l.* 2*s.*

MULTIPLICATION
OF
DECIMALS.

MULTIPLY as in common multiplication, and cut off as many in the product as there are decimals in both multiplier and multiplicand.

EXAMPLES.

$$\begin{array}{lr}
\text{Multiplicand} \ldots\ldots & 7.246 \\
\text{Multiplier} \ldots\ldots\ldots & 53.4 \\
\hline
 & 28984 \\
 & 21738 \\
 & 36230 \\
\hline
\text{Product} \ldots\ldots\ldots & 386.9364 \\
\hline
\end{array}$$

The multiplicand of the foregoing example may be expreſſed $7\frac{246}{1000} = \frac{7246}{1000}$; and the multiplier $53\frac{4}{10} = \frac{534}{10}$, *See page* 60. Then $\frac{7246}{1000} \times \frac{534}{10} = \frac{3869364}{10000} = 386\frac{9364}{10000} = 386.9364$. Hence the juſtneſs of the rule for cutting off the product appears; for it is very evident, that if the multiplicand and multiplier be expreſſed and multiplied like vulgar fractions, the denominator of the product will contain as many cyphers as are in both multiplicand and multiplier.

More EXAMPLES.

The product of 3.524 by 2.436 is 8.584464.

The product of .2057 by .315 is .0647955.

The product of .228 by .0034 is .0007752.

SUBTRACTION
OF
DECIMALS.

PLACE units under units, *&c.* as directed in addition, and then proceed as in common ſubtraction.

EXAMPLES.

From	723.424	From	627.002
Take	476.276	Take	477.725
	247.148		149.277

Subtract 74.375 from 81.628.
The remainder is 7.253.

Subtract 618.42 from 7482.6.
The remainder is 6864.18.

Subtract 74*l.* 15*s.* from 91*l.* .25.
The remainder is 16.5 = 16*l.* 10*s.*

DIVISION

OF

DECIMALS.

PERFORM the operation in the ſame manner as in common diviſion, and cut off ſo many in the quotient, that the decimals in the diviſor and quotient together may be equal in number to thoſe of the dividend.

For the product of the diviſor and quotient is equal to the dividend, and conſequently there muſt be as many cut off in the diviſor and quotient as in the dividend.

EXAMPLES.

I. When there are more decimals in the dividend than in the diviſor, cut off their difference in the quotient.

Divide 386.9364 by 53.4.
The quotient is 7.246.

Divide 9858.66618 by 27.83.
The quotient is 354.246.

Divide 842.6286 by 25.728.
The quotient is 32.7, and the remainder 13230.

When there is a remainder, cyphers may be joined to the dividend, and the diviſion continued until there is three or more decimals in the quotient.

II. When there are more decimals in the diviſor than in the dividend, add cyphers to the dividend equal to the difference, and the quotient will be a whole number, while theſe cyphers laſt; but if more be added, the ſame number of decimals will ariſe in the quotient.

Divide 73.2 by 6.326.

If two cyphers be added, the quotient will be 11, and if ſix be added, the quotient will be 11.5712.

Divide 364 by 2.34.

The quotient is 155, and if continued, it is 155.5+.

III. When there are not ſo many figures in the quotient as the rule requires to be cut off, prefix a cypher or cyphers.

Divide 8.88408 by 274.2.

The quotient is .0324.

Divide 1 by 282, and continue the quotient to ſix places.

The quotient is .003546.

ARITHMETIC.

BOOK III.

Practice, and Proportion.

PRACTICE.

PRACTICE is the moſt generally uſeful rule in Arithmetic, for by it the value of any quantity of goods is readily known.

The following caſes are all that can poſſibly occur. It will be a great advantage to the learner to have them all together. Examples to each are given afterwards.

I. If one pound be diviſible by the price, divide the given quantity by the number of times which the price is contained in one pound. The quotient is the anſwer in pounds, and the remainder, if any, is pounds; which reduce lower, and divide by the ſame diviſor.

For half a yard, when it occurs, add half the price of a yard. For one-eighth, or one-fourth, add the one-eighth or one-fourth of the price of one yard.

When $\frac{3}{4}$ or $\frac{7}{8}$ belong to the quantity, find the value for one yard more than the quantity, and ſubtract $\frac{1}{4}$ or $\frac{1}{8}$ of the price of one yard.

II. When a pound is not diviſible by the price, divide it into two ſuch parts, that the greater may be an equal part of a pound, and the leſſer an equal part of the greater.

Find the value of the given quantity for the greater part of the price by CASE I. and divide that value by the proportion between the two parts of the price. The amount of theſe two ſums is the total value of the given quantity.

III. The moſt conciſe method, ſometimes, is to divide the price into two ſuch parts, that each of them may be an even part of a pound. When it is found beſt to do this, the quantity muſt be divided by both the diviſors, and not the firſt value by the other diviſor. The amount of the two ſums, or lines, is the total value.

IV. There are a few prices, and to a ſkilful accountant, but a few, which require to be divided into three parts; the firſt an even part of a pound, the ſecond an even part of the firſt, and the leaſt an even part of the ſecond. The quantity muſt be divided by the firſt diviſor; the firſt value muſt be divided by the ſecond diviſor; and the ſecond value by the third diviſor. The ſum of theſe three values is the total one.

V. When the price is leſs than a ſhilling, the firſt diviſor muſt be a part of a ſhilling, and the total amount will be ſhillings, which muſt therefore be divided by 20.

VI. If the price be leſs than one penny, the firſt diviſor will be a part of a penny, and the total amount muſt be divided by 12 and 20.

VII. If the price be pounds only, multiply the quantity by the pounds; and if there be alſo ſhillings, *&c.* calculate for them by ſome of the preceding caſes.

VIII. When the price is very near an even part of a pound, compute the value of the quantity according to that even part by CASE I. then find the value of the quantity at the difference between the even part and the real price.

If the real price be greater than the even part, add the latter value to the former; but if the real price be leſſer, ſubtract.

The given price frequently approaches very near to ſome of the prices mentioned in the other CASES.—— When it does, compute for the price which it approaches to, and having found the total amount, find the value of the quantity at the difference between the two prices, which add to, or ſubtract from, the total amount, according to the preceding direction for that purpoſe.

IX. If the price be very near a pound, call the quantity pounds. Compute for the difference, the amount of which add to the quantity, if the price be more than a pound, but ſubtract if the price be leſs than a pound.

When the price is near any number of pounds, multiply the quantity by theſe pounds, and add or ſubtract the value of the difference.

X. When the quantity is ſmall, the price of different denominations, and not an even part of a pound, the ſhorteſt way generally is, to multiply the price by the quantity, or its component parts, as fully directed in Multiplication of Money. *See* page 26, and the four following.

XI. If the quantity conſiſts of different denominations, find the value of the integral, or whole part of it, by any of the preceding CASES; and compute for the inferior denominations by taking ſuch parts of the price as theſe denominations are of the principal one.

In theſe eleven CASES are comprehended rules of computation for all prices that can be given. The following remarks, however, will in ſome inſtances be uſeful.

Firſt, If there be, at an equal price, two quantities whoſe proportion is expreſſed by a whole number, and the value of the leſſer quantity known, the value of the greater is readily found by multiplying by the proportion. But if the value of the greater quantity be known, divide by the proportion.

Second, When two quantities are equal, and an integral, or whole number proportion between the prices, multiply the value found or given, for the leſſer price, by the proportion. But if the value for the greater price be known firſt, divide by the proportion.

Third, When there is an integral proportion between two quantities, and alſo between their prices, if the leſſer quantity have the leſſer price, multiply the value of the leſſer quantity by the product of the two proportions. But if the value of the greater quantity be firſt known, divide.

If the greater quantity has the leſſer price, and the value of that quantity be known, multiply by the proportion between the prices, and divide by the proportion between the quantities.

Fourth, If the quantity be 12, the anſwer will be as many ſhillings as the price conſiſts of pence. If the quantity be a multiple of 12, as 24, 36, *&c.* multiply the pence of the price by the proportion, and the product will be the anſwer in ſhillings.

Although the quantity ſhould neither be 12, nor any multiple of 12, it may be ſometimes proper to compute for as many twelves as it contains, and to find the value of the remainder by even parts.

Fifth, When the quantity is 240, the anſwer will be as many pounds as the price conſiſts of pence; if 480, the anſwer will be twice as many pounds as the pence conſiſts of pence, *&c.*

When the quantity is 20, the anſwer will be as many pounds as the price contains ſhillings. If the quantity be a multiple of 20, as 40, 60, *&c.* multiply the ſhillings of the price by the proportion.

			d.
If the quantity be	40	The *Anſwer* will be as many pounds as the price contains -	6
	60		4
	80		3
	120		2
	160		$1\frac{1}{2}$
	192		$1\frac{1}{4}$

Sixth, If the price be any of thoſe in TABLE II. multiply the quantity by the numerator, and divide by the denominator, the quotient is the value in pounds.

The operation will be much ſhorter, if the quantity be diviſible by the denominator. If it is, multiply their quotient by the numerator.

Seventh, If the price be reduced into its leaſt given denomination, and then multiplied by the quantity, the product will be the value in the denomination into which the price was reduced, and may be turned into pounds by Reduction Aſcending.

The Uſe of the three following Tables.

TABLE I.

The upper part of the firſt column contains all the prices which are even parts of a pound; and the ſecond ſhews how often they are contained in a pound; thus, 10*s.* is contained 2 times in a pound; 6*s.* 8*d.* is contained 3 times; 5*s.* four times, &*c.*

The third and fourth columns contain the even parts of 10*s.* for inſtance, 5*s.* is contained 2 times in 10*s.*—1*s.* 3*d.* is contained 8 times in 10*s.*

The other columns contain the even parts of 6*s.* 8*d.* 5*s.*—4*s.* &*c.*

If the price be 12*s.* 6*d.* it muſt, by CASE II. be taken at twice. By the two firſt columns 10*s.* are contained 2 times in a pound; and by the third and fourth columns 2*s.* 6*d.* is contained 4 times in 10*s.* The other tables are applied in a ſimilar manner.

TABLE II.

This table contains a number of prices, which are not even, but fractional parts of a pound.

TABLE III.

This table is for avoirdupoiſe weight, and is on the the ſame plan for quarters and pounds weight as TABLE I. is for ſhillings and pence.

Thus, by the two firſt columns 2*qr.* are contained 2 times in 1*c.*; again, 16*lb.* are contained 7 times in 1*c.* *&c.*

By the third and fourth columns 21*lb.* are contained 4 times in 3*qr.* alſo, 7*lb.* are contained 12 times in 3*qr.* *&c.*

TABLE I.

Parts of a Pound.	Diviſors.	Parts of 10*s*.	Diviſors.	Parts of 6*s*. 8*d*.	Diviſors.	Parts of 5*s*.	Diviſors.	Parts of 4*s*.	Diviſors.
s. *d*.		*s*. *d*.		*s*. *d*.		*s*. *d*.		*s*. *d*.	
10 0	2	5 0	2	3 4	2	2 6	2	2 0	2
6 8	3	3 4	3	1 8	4	1 8	3	1 4	3
5 0	4	2 6	4	1 4	5	1 3	4	1 0	4
4 0	5	2 0	5	10	8	1 0	5	8	6
3 4	6	1 8	6	8	10	10	6	6	8
2 6	8	1 3	8	4	20	7½	8	4	12
2 0	10	1 0	10			6	10		
1 8	12	10	12			5	12		
1 0	20	6	20			3	20		

Parts of 3*s*. 4*d*.	Diviſors.	Parts of 2*s*. 6*d*.	Diviſors.	Parts of 2*s*.	Diviſors.	Pts. of 1*s*. 8*d*.	Diviſors.	Parts of 1*s*.	Diviſors.
s. *d*.		*s*. *d*.		*s*. *d*.		*d*.		*d*.	
1 8	2	1 3	2	1 0	2	10	2	6	2
10	4	10	3	8	3	5	4	4	3
8	5	7½	4	6	4	4	5	3	4
5	8	6	5	4	6	2½	8	2	6
4	10	5	6	3	8	2	10	1½	8
2	20	3¾	8	2	12	1	20	1	12
		3	10						
		2½	12						
		1½	20						

TABLE II.

s. *d.*	Fractional Parts of a Pound.	*s.* *d.*	Fractional Parts of a Pound.	*s.* *d.*	Fractional Parts of a Pound.	*s.* *d.*	Fractional Parts of a Pound.
2 8	$\frac{2}{15}$	8 0	$\frac{2}{5}$	11 8	$\frac{7}{12}$	14 8	$\frac{11}{15}$
3 9	$\frac{3}{16}$	8 4	$\frac{5}{12}$	12 0	$\frac{3}{5}$	15 0	$\frac{3}{4}$
5 4	$\frac{4}{15}$	8 9	$\frac{7}{16}$	12 6	$\frac{5}{8}$	16 0	$\frac{4}{5}$
6 0	$\frac{3}{10}$	9 4	$\frac{7}{15}$	13 4	$\frac{2}{3}$	16 8	$\frac{5}{6}$
6 3	$\frac{5}{16}$	10 8	$\frac{8}{15}$	13 9	$\frac{11}{16}$	17 6	$\frac{7}{8}$
7 6	$\frac{3}{8}$	11 3	$\frac{9}{16}$	14 0	$\frac{7}{10}$	18 4	$\frac{11}{12}$

TABLE III.

Parts of a *Cwt.*	Divisors.	Parts of 3*qrs.*	Divisors.	Parts of 2*qrs.*	Divisors.	Parts of 1*qr.*	Divisors.
qr. *lb.*		*qr.* *lb.*		*qr.* *lb.*		*lb.* *oz.*	
2 0	2	1 0	3	1 0	2	14 0	2
1 0	4	21	4	14	4	7 0	4
16	7	14	6	8	7	4 0	7
14	8	12	7	7	8	3 8	8
8	14	10½	8	4	14	2 0	14
7	16	7	12	3½	16	1 12	16

EXAMPLES *to* CASE I.

How much do 536 yds. come to at 10*s.* per yd.?

It is evident that the anſwer will be half as many pounds as yards, the price 10*s.* being contained twice in a pound. Half of any number is obtained by dividing by 2.

s.		
10	2	536
		268*l.* *Anſwer.*

How much do 736 yds. come to at 5*s.* each?

Anſwer 184*l.*

How much do 4957 gallons come to at 3*s.* 4*d.* per gallon?

Anſwer 826*l.* 3*s.* 4*d.*

Required the value of 435 yards at 1*s.* 8*d* per yard?

Anſwer 36*l.* 5*s.*

How much do 825 gals. come to at 6*s.* 8*d.* per gal.?

It is evident that the anſwer will be in pounds one third of the number of gallons, as 6*s.* 8*d.* is contained thrice in a pound. One-third of any number is found by dividing by 3.

s. *d.*		
6 8	3	825
		275*l.* *Anſwer.*

How much do 935 yds. come to at 4*s.* per yd.?

Anſwer 187*l.*

How much do 6738 gallons come to at 2*s.* 6*d.* per gallon?

Anſwer 842*l.* 5*s.*

Required the value of 312 yards at 1*s.* per yard?

Anſwer 15*l.* 12*s.*

EXAMPLES *to* CASE II.

Required the value of 432 yards, at 12*s.* 6*d.* per yard?

The two moſt proper parts to divide the price into, are 10*s.* and 2*s.* 6*d.* the former being contained twice in a pound, and 2*s.* 6*d.* is contained four times in 10*s.* *See the proper columns in* TABLE I.

s. *d.*		
10 0	2	432
2 6	4	216
		54
		270*l.* *Anſwer.*

At 10*s.* the value is 216*l.* and at 2*s.* 6*d.* it is evidently one-fourth thereof, *viz.* 54*l.* and conſequently at 12*s.* 6*d.* the value is 270*l.*

How much do 524 gallons come to, at 5*s.* 10*d.* per gallon?

Anſwer 152*l.* 16*s.* 8*d.*

Required the value of 576 yards, at 8*s.* 4*d.* per yard?

The two moſt proper parts to divide 8*s.* 4*d.* into, are 6*s.* 8*d.* and 1*s.* 8*d.* the former being contained thrice in a pound, and 1*s.* 8*d.* four times in 6*s.* 8*d.* *See the proper columns in* TABLE I.

s. *d.*		
6 8	3	576
1 8	4	192
		48
		240*l.* *Anſwer.*

At 6*s.* 8*d.* the value is 192*l.* and at 1*s.* 8. it is evidently one-fourth thereof, *viz.* 48*l.* conſequently at 8*s.* 4*d.* the value is 240*l.*

How much do 475 yards come to, at 4*s.* 6*d.* per yard?

Anſwer 106*l.* 17*s.* 6*d.*

How much do 408 yards come to, at 4*s.* 2*d.* per yard?

Anſwer 85*l.*

How much do 708 yards come to, at 5*s.* 7½*d.* per yard?

Anſwer 199*l.* 2*s.* 6*d.*

How much do 723 yards come to, at 3*s.* 1½*d.* per yard?

Anſwer 112*l.* 19*s.* 4½*d.**

How much do 436 yards come to, at 2*s.* 9¾*d.* per yard?

Anſwer 61*l.* 6*s.* 3*d.*

EXAMPLES *to* CASE III.

Required the value of 84 gallons, at 6*s.* 6*d.* per gallon?

s. *d.*		
4 0	5	84
2 6	8	16 16
		10 10
		27 6

Required the value of 78 yards, at 9*s.* 2*d.* per yard?

Anſwer 35*l.* 15*s.*

Required the value of 712 yards, at 8*s.* 8*d.* per yard?

Anſwer 308*l.* 10*s.* 8*d.*

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* The parts of the price in this example are 2*s.* 6*d.* and 7½*d.* the latter one-fourth of the former. The value for 2*s.* 6*d.* is 90*l.* 7*s.* 6*d.* In dividing it by 4, there remains 2*l.* or 40*s.* to which add the 7*s.* When 47 is divided by 4, there remains 3*s.* or 36*d.* to which add the 6*d.* When 42 is divided by 4, the remainder is 2*d.* or 8 farthings, wherein 4 is contained twice, therefore the value for 7½*d.* is 22*l.* 11*s.* 10½*d.*

The prices in this caſe can only be ſo divided, that each part may be contained exactly in a pound, and therefore the quantity is divided by both diviſors, and not the firſt value by the ſecond diviſor, as in the preceding caſe.

How much do $56\frac{1}{2}$ yds. come to, at 7*s.* 4*d.* per yard?

Anſwer 20*l.* 14*s.* 4*d.*

Required the value of 148 gallons, at 3*s.* 6*d.* per gallon?

Anſwer 25*l.* 18*s.*

How much do $74\frac{1}{4}$ gallons come to, at 10*s.* 8*d.* per gallon?

Anſwer 39*l.* 12*s.*

EXAMPLES *to* CASE IV.

How much do 418 yds. come to, at 4*s.* $7\frac{1}{2}$*d.* per yard?

s. *d.*		
4 0	5	418
6	8	83 12
$1\frac{1}{2}$	4	10 9
		2 12 3
		96 13 3

Required the value of $75\frac{7}{8}$ gallons, at 11*s.* 6*d.* per gallon?

Anſwer 43*l.* 12*s.* $6\frac{3}{4}$*d.*

How much do 470 gallons come to, at 5*s.* 8*d.* per gallon?

s. *d.*		
5 0	4	470
6	10	117 10
2	3	11 15
		3 18 4
		133 3 4

Required the value of 92 yards, at 2*s.* $4\frac{1}{2}$*d.* per yard?

Anſwer 10*l.* 18*s.* 6*d.*

EXAMPLES to CASE V.

Required the value of 432 yards, at 6*d.* per yard?

d.		
6	2	432
	2,0	21,6 ſhillings.
		10 16

Required the value of 76 pounds, at 3*d.* per pound?

Anſwer 19*s.*

Required the value of 58 yards, at 7 ½*d.* per yard?

Anſwer 1*l.* 16*s.* 3*d.*

Required the value of 512 yards, at 4*d.* per yard?

d.			
4	3	512	
	2,0	17,0	8*d.*
		8 10	8

Required the value of 216 pair, at 2*d.* per pair?

Anſwer 1*l.* 16*s.*

Required the value of 232 pounds, at 3¾*d.* per pound?

Anſwer 3*l.* 12*s.* 6*d.*

EXAMPLES to CASE VI.

How much do 432 yds. come to, at ¼*d.* per yard?

¼	4	432
	12	108 pence.
		9 ſhillings.

How much do 38 yards come to, at ½*d.* per yard?

½	2	38
	12	19 pence.
		1 7

EXAMPLES *to* CASE VII.

How much do 384 acres come to, at 34*l.* per acre?

Anſwer 13056*l.*

How much do 43 ſheep come to, at 2*l.* 7*s.* 6*d.* each?

s. *d.*				
5 0	4	43		
		2		
		86		
2 6	2	10	15	
		5	7	6
		102	2	6

Be always particularly careful not to divide the product of the quantity and pounds, inſtead of the quantity itſelf.

How much do 250 hundred weight come to, at 8*l.* per hundred weight?

Anſwer 2000*l.*

How much do 36 oxen come to, at 12*l.* 4*s.* 8*d.* each?

Anſwer 440*l.* 8*s.*

How much do 284 weeks come to, at 7*l.* 11*s.* 8*d.* each?

Anſwer 2153*l.* 13*s.* 4*d.*

How much do 532 hundred weight come to, at 5*l.* 3*s.* 9½*d.* per cwt?

Anſwer 2760*l.* 17*s.* 2*d.*

EXAMPLES *to* CASE VIII.

How much do 42 yards come to, at 10*s.* 1*d.* per yard?

How much do 72 yards come to, at 6*s.* 8½*d.* per yard?

The price being only a little above 10*s.* find the value for 10*s.* and add 42*d.* or 3*s.* 6*d.* for the 1*d.* per yard above the 10*s.*

s.		
10	2	42
		21 0
Add		3 6
		21 3 6

How much do 112 gallons come to, at 5*s.* 2*d.* per gallon?

Anſwer 28*l.* 18*s.* 8*d.*

Required the value of 224 yards, at 2*s.* 6½*d.* per yard?

Anſwer 28*l.* 9*s.* 4*d.*

The price being only ½*d.* above 6*s.* 8*d.* find the value for 6*s.* 8*d.* and add 72 half-pence, or 36*d.* or 3*s.* for the ½*d.* per yard above 6*s.* 8*d.*

s. d.		
6 8	3	72
		24 0
Add		3
		24 3

How much do 64 ⅛ yds. come to, at 3*s.* 5½*d.* per yard?

Anſwer 11*l.* 1*s.* 9*d.* $\frac{3}{16}$

Required the value of 60 dozen, at 2*s.* 0¼*d.* per dozen?

Anſwer 6*l.* 1*s.* 3*d.*

In the four following examples the real price is leſſer than the even part, and therefore the value of the difference muſt be ſubtracted.

How much do 72 yards come to, at 9*s.* 11½*d.* per yard?

Anſwer 35*l.* 17*s.*

How much do 548 gallons come to, at 6*s.* 7*d.* per gallon?

Anſwer 180*l.* 7*s.* 8*d.*

How much do 68 yards come to, at 4*s.* 11¾*d.* per yard?

Anſwer 16*l.* 18*s.* 7*d.*

How much do 412 yards come to, at 3*s.* 3*d.* per yard?

Anſwer 66*l.* 19*s.*

In the four following queſtions, the prices are very near thoſe belonging to the ſecond caſe.

How much do 72 gallons come to, at 4*s.* 9*d.* per gallon?

s. *d.*				
4 0	5	72		
8	6	14	8	
		2	8	
Add		·	6	
		17	2	

How much do 208 yards come to, at 3*s.* 9½*d.* per yard?

s. *d.*				
3 4	6	208		
5	8	34	13	4
		4	6	8
Add		·	8	8
		39	8	8

Required the value of 265 yards, at 4*s.* 5½*d.* per yard?

Anſwer 59*l.* 1*s.* 5½*d.*

Required the value of 178 gallons, at 5*s.* 7*d.* per gallon?

Anſwer 49*l.* 13*s.* 10*d.*

In the four following examples the prices are very near thoſe mentioned in the third caſe.

Required the value of 152½ yards, at 9*s.* 0½*d.* per yard?

Anſwer 68*l.* 18*s.* 10¼*d.*

Required the value of 200 gallons, at 8*s.* 8¾*d.* per gallon?

Anſwer 87*l.* 5*s.* 10*d.*

How much do 222 gallons come to, at 6*s.* 4¾*d.* per gallon?

Anſwer 70*l.* 19*s.* 10½*d.*

How much do 408 yards come to, at 3*s.* 7*d.* each?

Anſwer 73*l.* 2*s.*

EXAMPLES. *to* CASE IX.

How much do 46 yards come to, at 19*s.* 6*d.* per yard?

	£	*s.*	
	46	0	
Subtract	1	3	
£	44	17	*Anſwer.*

How much do 84 gallons come to, at 19*s.* 11*d.* per gallon?

Anſwer 83*l.* 13*s.*

Required the value of 53*c.* at 4*l.* 17*s.* 6*d.* per *cwt.*?

s. d.				
2 6	8	53		
		5		
		265	0	0
Sub.		6	12	6
		258	7	6

How much do 44 acres come to, at 1*l.* 0*s.* 3*d.* per acre?

	£	*s.*	
	44	0	
Add		11	
£	44	11	*Anſwer.*

How much do 47 yards come to, at 1*l.* 0*s.* 6*d.* per yard?

Anſwer 48*l.* 3*s.* 6*d.*

Required the value of 428 quarters, at 7*l.* 18*s.* 4*d.* per quarter?

s. d.				
1 8	12	428		
		8		
		3424	0	0
Sub.		35	13	4
		3388	6	8

How much do 88 ½ acres come to, at 24*l.* 19*s.* per acre?

Anſwer 2208*l.* 1*s.* 6*d.*

How much do 75 ¼ acres come to, at 19*l.* 19*s.* per acre?

Anſwer 1501*l.* 4*s.* 9*d.*

EXAMPLES *to* CASE X.

A queſtion or two will be ſufficient in this caſe, as a great variety of examples of this kind are contained in the 26*th.* and the four following pages.

How much do 56 ½ yards come to, at 6*s.* 3½*d.* per yard?

Anſwer 17*l.* 15*s.* 5¾*d.*

How much do 41 ¾ gallons come to, at 7*s.* 6¾*d.* per gallon?

Anſwer 15*l.* 15*s.* 8¾*d.*

EXAMPLES *to* CASE XI.

Required the value of 7*c.* 2*qr.* 21*lb.* at 6*l.* 9*s.* 4*d.* per cwt.?

qr.	*lb.*				
2	0	2	6	9	4
					7
			45	5	4
	14	4	3	4	8
	7	2		16	2
				8	1
			49	14	3

Required the value of 4*c.* 3*qr.* 14*lb.* at 3*l.* 13*s.* 4*d.* per hundred weight?

qr.	*lb.*				
2	0	2	3	13	4
					4
			14	13	4
1	0	2	1	16	8
	14	2		18	4
				9	2
			17	17	6

The price of 1 hundred weight being multiplied by 7, the product 45*l.* 5*s.* 4*d.* is the value of 7*c.* The value of 2*qr.* is found to be 3*l.* 4*s.* 8*d.* by dividing the price of 1*c.* by 2, as 2*qr.* is contained twice in 1*c.* The value of 14*lb.* is known by dividing 3*l.* 4*s.* 8*d.* by 4, for 14*lb.* is contained 4 times in 2*qr.* and the remaining 7*lb.* being contained twice in 14*lb.* the value of it is known by dividing 16*s.* 2*d.* by 2.

How much do 72*a.* 3*r.* 25*p.* of land come to, at 2*l.* 13*s.* 4d. per acre?

Anſwer 194*l.* 8*s.* 4*d.*

How much do 4*c.* 2*qr.* 22*lb.* 8*oz.* come to, at 18*s.* 8*d.* per hundred weight?

Anſwer 4*l.* 7*s.* 9*d.*

How much do 7*c.* 2*qr.* 18*lb.* come to, at 8*l.* 6*s.* 2*d.* per hundred weight?

Anſwer 63*l.* 12*s.* 11½*d.*

How much do 13*lb.* 7*oz.* 10*dwt.* troy come to, at 6*l.* 8*s.* per pound?

Anſwer 87*l.* 4*s.*

How much do 54*a.* 2*r.* 26*p.* of land come to, at 8*l.* 16*s.* per acre?

Anſwer 481*l.* 0*s.* 7*d.* $\frac{1}{5}$

EXAMPLES *to the* FIRST REMARK.

The value of 48 yards at 2*s.* 4*d.* is 5*l.* 12*s.* required thence the value of 144 yards at the ſame price?

The proportion between 48 and 144 is 3, therefore multiply 5*l.* 12*s.* by 3, and the product 16*l.* 16*s.* is the anſwer.

The value of 22 yards at 3*s.* 2½*d.* is 3*l.* 10*s.* 7*d.* required thence the value of 154 yards at the ſame price?

The proportion between 22 and 154 is 7, therefore multiply 3*l.* 10*s.* 7*d.* by 7, and the product 24*l.* 14*s.* 1*d.* is the anſwer.

The value of 12 yards at 6*s.* 3*d.* is 3*l.* 15*s.* required the value of 72 yards at the ſame price?

Anſwer 22*l.* 10*s.*

The value of 165 yards at 4*s.* 3*d.* per yard is 35*l.* 1*s.* 3*d.* required the value of 33 yards at the ſame price?

The proportion is 5, whereby 35*l.* 1*s.* 3*d.* being divided, the quotient 7*l.* 0*s.* 3*d.* is the anſwer.

The value of 18 gallons at 5*s.* 4$\frac{3}{4}$*d.* is 4*l.* 17*s.* 1$\frac{1}{2}$*d.* required the value of 126 yards at the ſame price?

Anſwer 33*l.* 19*s.* 10$\frac{1}{2}$*d.*

The value of 16 yards at 10$\frac{1}{4}$*d.* is 13*s.* 8*d.* required the value of 133 yards at the ſame price?

The neareſt number to 133, to which 16 has an integral proportion, is 128, and the remaining 5 yards is readily computed for. The total value is 5*l.* 13*s.* 7$\frac{1}{4}$*d.*

EXAMPLES *to the* SECOND REMARK.

At 3*s.* 8*d.* per yard 42 yards come to 7*l.* 14*s.* required the value of the ſame number of yards at 14*s.* 8*d.* each?

Anſwer 30*l.* 16*s.*

At 2*s.* 8*d.* per yard 15 yards come to 2*l.* how much will 15 yards come to at 18*s.* 8*d.* per yard?

Anſwer 14*l.*

At 15*s.* 5*d.* per yard 34 yards come to 26*l.* 4*s.* 2*d.* required the value of the ſame number of yards at 3*s.* 1*d.* per yard?

Anſwer 5*l.* 4*s.* 10*d.*

At 1*l.* 5*s.* 10*d.* per yard 63 yards come to 81*l.* 7*s.* 6*d.* how much will 63 yards come to at 5*s.* 2*d.* each?

Anſwer 16*l.* 5*s.* 6*d.*

EXAMPLES *to the* THIRD REMARK.

The value of 42 yards at 3*s*. 5*d*. is 7*l*. 3*s*. 6*d*. required the value of 126 yards at 6*s*. 10*d*.?

3 × 2 = 6, the product of the two proportions.

7*l*. 3*s*. 6*d*. × 6 = 43*l*. 1*s*. the Anſwer.

The value of 88 yards at 7*s*. 4*d*. is 32*l*. 5*s*. 4*d*. required the value of 22 yards at 3*s*. 8*d*.?

4 × 2 = 8, the product of the two proportions.

32*l*. 5*s*. 4*d*. ÷ 8 = 4*l*. 0*s*. 8*d*. the Anſwer.

EXAMPLES *to the* FOURTH REMARK.

How much do 12 gallons come to, at 7*s*. 6*d*. per gallon?

Anſwer 90*s*. = 4*l*. 10*s*.

Required the value of 60 yards, at 2*s*. 7*d*. per yard?

Anſwer 7*l*. 15*s*. for 31 × 5 = 155*s*. = 7*l*. 15*s*.

How much do 98 yds. come to, at 4*s*. 8*d*. per yard?

56 × 8 = 448*s*. = 22*l*. 8*s*. for 96 yards, to which 9*s*. 4*d*. for 2 yards being added, the ſum 22*l*. 17*s*. 4*d*. is the Anſwer.

How much do 24 yards come to, at 8*s*. 4*d*. per yd.?

Anſwer 10*l*. for 100 × 2 = 200*s*. = 10*l*.

Required the value of 84 yards, at 4*s*. 3½*d*. per yard?

Anſwer 18*l*. 0*s*. 6*d*. for 51·5 × 7 = 360·5 = 18*l*. 0*s*. 6*d*.

How much do 110½ yds. come to, at 5*s*. 5½*d* per yd.?

65·5 × 9 = 589·5 =

		29*l*.	9*s*.	6*d*.
2 yards	-	·	10	11
½ yard	-	·	2	8¾
Anſwer	-	30	3	1¾

EXAMPLES *to the* FIFTH REMARK.

How much do 240 gallons come to, at 6*s.* 4*d.* per gallon?

Anſwer 76*l.*

How much do 40 yards come to, at 17*s.* per yard?

Anſwer 34*l.*

How much do 120 pair come to, at 5*s.* 6*d.* per pair?

Anſwer 33*l.* for 11 × 3 = 33.

Required the value of 324 pounds, at 4½*d.* per pound?

	£	*s.*	*d.*
320*lb.*	6	0	0
4*lb.*	.	1	6
Anſwer	6	1	6

How much do 480 gallons come to, at 7*s.* 4*d.* per gallon?

Anſwer 176*l.*

How much do 40 yards come to, at 3*s.* 6*d.* per yard?

Anſwer 7*l.*

How much do 180 gallons come to, at 4*d.* per gallon?

Anſwer 3*l.*

Required the value of 196 yards, at 1$\frac{1}{4}$ each?

	£	*s.*	*d.*
192 yards	1	0	0
4 yards	0	0	5
Anſwer £	1	0	5

EXAMPLES *to the* SIXTH REMARK.

How much do 64 yards come to, at 2*s.* 8*d.* per yard?

$\frac{64 \times 2}{15}$ = 8*l.* 10*s.* 8*d.*

Required the value of 132 gallons, at 11*s.* 8*d.* per gallon?

$\frac{132}{12} \times 7$ = 77*l.*

How much do 105 gallons come to, at 5*s.* 4*d.* per gallon?

$\frac{105}{15} \times 4$ = 28*l.*

Required the value of 43 gallons, at 13*s.* 4*d.* per gallon?

$\frac{43 \times 2}{3}$ = 28*l.* 13*s.* 4*d.*

EXAMPLES *to the* SEVENTH REMARK.

At 7*s.* 3*d.* what do 62 yards come to?

7*s.* 3*d.* = 87*d.* Then 87 × 62 = 5394*d.* = 22*l.* 9*s.* 6*d,*

At 12*s.* what do 52 yards come to?

$\frac{32 \times 12}{20}$ = 19*l.* 4*s.*

At 4*s.* 2½*d.* what do 54½ yards come to?

4*s.* 2½*d.* = 202*qr.* Then 202 × 54.5 = 11009*qr.* = 11*l.* 9*s.* 4¼*d.*

At 11*s.* what do 412 yards come to?

Anſwer 226*l.* 12*s.*

If a perſon receives 4*s.* 2*d.* per day, how much will he receive from June 9*th*, to September 12*th*, both incluſive?

Anſwer 20*l.*

How much do 4*c.* 2*qr.* 8*lb.* come to, at 10*d.* per pound?

4*c.* 2*qr.* 8*lb.* = 512*lb.* by reduction. Then 512 × 10 = 5120*d.* = 21*l.* 6*s.* 8*d.*

The quantity and the value are ſometimes given to find the price of one.

Divide the total value, reduced if neceſſary, by the quantity, and the quotient is the value of 1.

If 328 hundred weight be bought for 779*l.* how much is that for 1 hundred weight?

Anſwer 2*l.* 7*s.* 6*d.*

When 64 yards coſt 8*l.* 10*s.* 8*d.* what is the price of 1 yard?

Anſwer 2*s.* 8*d.*

DIRECT PROPORTION;

OR, THE

GOLDEN RULE.

WHEN, of four numbers, there is the ſame proportion between the two firſt as between the two laſt, theſe numbers are in direct proportion.

12, 6, 32, 16, are in direct proportion; for the firſt is twice the ſecond, and the third is twice the fourth; 2 being the proportion, or ratio.

5, 25, 8, 40, are in direct proportion, (÷) for the firſt is one-fifth of the ſecond, and the third is one-fifth of the fourth; the ratio being 5.

The firſt and third are called antecedents, and the ſecond and fourth conſequents.

The meaning of the proportional ſeries 8 ·· 24 : : 13 ·· 39, is, that as 8 is to 24, ſo is 13 to 39.——*See Characters*, page 1:

The firſt and laſt terms are the extremes, and the ſecond and third are the means.

PRO-

PROPOSITION I.

When four numbers are in direct proportion, the product of the two extremes is equal to the product of the two means, that is, in the ſeries $12 \cdot\cdot 36 :: 8 \cdot\cdot 24$, it will be $12 \times 24 = 36 \times 8$.

The proportion, or ratio, between two numbers being the quotient of their diviſion, $\frac{36}{12}$, is an expreſſion for the ratio between the two firſt, and $\frac{24}{8}$ for the ratio between the two laſt. Theſe are equal by the definition of direct proportionals, that is $\frac{36}{12} = \frac{24}{8}$.

By multiplication of fractions $12 \times 24 = 36 \times 8$; and it is very evident that the reſult will be ſimilar with reſpect to all other direct proportionals.

Another method of demonſtrating the propoſition:

Let A repreſent the firſt number, r the ratio; B the third number, then $A \times r$ will be the ſecond number, and $B \times r$ the fourth, and the ſeries, or four proportionals, will be $A \cdot\cdot \overline{A \times r} :: B \cdot\cdot \overline{B \times r}$, and conſequently $A \times B \times r$, the product of the two extremes, is equal to $A \times r \times B$, the product of the two means; for the multipliers which conſtitute one product, are the very ſame as the multipliers which conſtitute the other product, that is, the firſt number, the ratio, and third number, are the components of each product.

PROPOSITION II.

When three numbers are given, a fourth to form a direct proportion may be found by any of the three following methods:

I. Multiply the ſecond by the third, and divide the product by the firſt. The quotient is the fourth, or required number.

II. Divide the ſecond or third by the firſt, and multiply the other by the quotient, the product is the anſwer.

III. Divide the firſt by the ſecond or third, and divide the other by the quotient, and the reſult will be the anſwer.

Demonſtration of theſe Rules.

Let A, B, C, and D, repreſent four numbers in ∺ then,

$$B \times C = A \times D, \text{ by Prop. I.}$$

Divide by A, and the reſult will be $\frac{B \times C}{A} = D$, the fourth number, which is a demonſtration of the firſt rule.

The equation $\frac{B \times C}{A} = D$ may be expreſſed $\frac{B}{A} \times C = D$; or $\frac{C}{A} \times B = D$, which demonſtrates the ſecond rule.

When C is actually divided by $\frac{A}{B}$ the quotient is $\frac{B \times C}{A}$ and when B is actually divided by $\frac{A}{C}$ the quotient is $\frac{B \times C}{A}$ which is a demonſtration of the third rule, for the final reſult is evidently the ſame by that rule as by the firſt.

EXAMPLES *to this* PROPOSITION.

Given three numbers, 16, 64, and 80, to find a fourth proportional.

Firſt Method. 16 .. 64 :: 80 .. 320, the anſwer, for $\frac{64 \times 80}{16} = 320$.

Second Method. $\frac{64}{16} \times 80 = 320$; or $\frac{80}{16} \times 64 = 320$.

Third Method. $\frac{16}{64} = \frac{1}{4}$, then $80 \div \frac{1}{4} = 320$; or $\frac{16}{80} = \frac{1}{5}$, then $64 \div \frac{1}{5} = 320$.

To divide a whole number by a fraction, whoſe numerator is 1, is only to multiply the whole number by the denominator.

When this excellent propoſition is applied to accounts, place or ſtate the given numbers ſo, that the firſt and third may be of the ſame kind and denomination with each other, the third being that to which, in the queſtion, the anſwer is demanded. The other number muſt neceſſarily be the ſecond, and the fourth will always be of the ſame kind and denomination with it.

When they are properly ſtated, find the anſwer, or fourth proportional, by any of the three methods.

EXAMPLES.

If 34 yards coſt 16*l.* how much will 51 yards coſt?

In this example the numbers are already in their proper order, the firſt and third being both yards of the ſame kind, and the third being that to which an anſwer is demanded; therefore,

As 34*y.* ·· 16*l.* : : 51*y.* ·· 24*l.* for $\frac{51 \times 16}{24} = 24l.$ by Method I.

When 32 yards coſt 18*l.* how much will 144 yards come to?

As 32*y.* ·· 18*l.* : : 144*y.* ·· 81*l.* for $\frac{144 \times 18}{32} = 81l.$

If 72 yards of damaſk coſt 34*l.* how much will 441 yards coſt?

As 72*y.* ·· 34*l.* : : 441*y.* ·· 208*l.* 5*s.* for $\frac{441 \times 34}{72}$ $= 208l.\, \frac{1}{4} = 208l.\, 5s.$

If 96 gallons coſt 15*l.* how much will 324 gallons coſt?

As 96*g.* ·· 15*l.* : : 324*g.* ·· 50*l.* 12*s.* 6*d.* for $\frac{324 \times 15}{96} = 50l.\, \frac{5}{8} = 50l.\, 12s.\, 6d.$

When 28 gallons are ſold for 15*l.* 8*s.* how much muſt 73 gallons be ſold for?

By reduction 15*l.* 8*s.* = 308*s.* then, as 28*g.* ·· 308*s.* :: 73*g.* ·· 803*s.* for, by Method ſecond, $\frac{308}{28} \times 73$ = 803*s.* which divide by 20, and the anſwer is 40*l.* 3*s.*

It will often render the operation much more conciſe, if the firſt and ſecond, or firſt and third terms of the ſtating be divided by their greateſt common diviſor.

When 64 yards coſt 4*l.* 8*s.* how much will 75 yards come to?

As 64*y.* ·· 88*s.* :: 75*y.* ·· the anſwer.

Or by dividing the two firſt terms by 8, it will be, as 8 ·· 11 :: 75 ·· 103*s.* $\frac{1}{8}$ = 5*l.* 3*s.* 1$\frac{1}{2}$*d.*

When 36 gallons coſt 24*l.* 12*s.* how much will 85 gallons coſt?

Anſwer 58*l.* 1*s.* 8*d.*

If 72 yards are ſold for 26*l.* how many yards will 65*l.* buy?

The two firſt numbers muſt not be ſtated in the order they are given in the queſtion, but thus,

As 26*l.* ·· 72*y.* :: 65*l.* ·· the anſwer.

Or, as 2 ·· 72 :: 5 ·· 180 yards.

When 17 yards coſt 3*l.* 7*s.* 6*d.* how many yards will 8*l.* 8*s.* 9*d.* buy?

Anſwer 42 ½ yards.

If 31 ½ yards are ſold for 19*l.* 12*s.* how much muſt 47 ¼ yards be ſold for?

As $\overset{y.}{31.5} \cdot\cdot \overset{l.}{19.6} :: \overset{y.}{47.25} \cdot\cdot \overset{l.}{29.4}$, for $\frac{47.25}{31.5} \times 19.6 = 29.4 =$ 29*l.* 8*s.*

If 25 ½ yards are ſold for 9*l.* 11*s.* 3*d.* how much muſt 38 ¼ yards be ſold for?

Anſwer 14*l.* 6*s.* 10½*d.*

If 1 hundred weight be ſold for 3*l.* 10*s.* how much muſt 2*c.* 3*qr.* 14*lb.* be ſold for?

In 1*c.* there are 112*lb.* and 322*lb.* in 2*c.* 3*qr.* 14*lb.* therefore,

As 112*lb.* ·· 70*s.* :: 322*lb.* ·· the anſwer.

Or, as 8*lb.* ·· 5*s.* :: 322*lb.* ·· 201 ¼, for $\frac{322 \times 5}{8}$ = 201 ¼ = 10*l.* 1*s.* 3*d.*

When 3*c.* 2*qr.* 8*lb.* coſt 932*l.* 12*s.* 6*d.* how much will 7*c.* 3*qr.* 12*lb.* coſt?

Anſwer 2051*l.* 15*s.* 6*d.*

Required the value of 24 pieces of linen, each piece containing 30 yards, if 64 yards coſt 9*l.*?

24 ×

$$24 \times 30 = 720.$$

As 64*y*. ·· 9*l*. :: 720*y*. ·· the anſwer.

Or, as 4*y*. ·· 9*l*. :: 45*y*. ·· 101*l*. 5*s*.

If 32 gallons of wine coſt 17*l*. 6*s*. 8*d*. how much will 56 gallons coſt?

There is no occaſion, in this example, to reduce the given value into pence, for the firſt and third numbers, by a common diviſor, become each leſs than 13.

As 32 ·· 17*l*. 6*s*. 8*d*. :: 56 ·· the anſwer.

Or. as 4 ·· 17*l*. 6*s*. 8*d*. :: 7 ·· 30*l*. 6*s*. 8*d*.

The anſwer to a queſtion is ſometimes readily found thus: Divide the third by the firſt, even when it is certain that there will be a remainder. Multiply the ſecond term by the quotient, and add ſuch parts of the ſecond as the remainder is of the firſt.

When 64 yards are ſold for 42*l*. 13*s*. 6*d*. how much will 272 yards be ſold for?

$\frac{272}{64} = 4$, and remainder 16, which is contained four times in 64.

16	4	42	13	6	
				4	
		170	14	0	
		10	13	$4\frac{1}{2}$	
		181	7	$4\frac{1}{2}$	*Anſwer.*

When 42 gallons coſt 15*l.* 10*s.* 4*d.* how much will 108 gallons coſt?

$\frac{108}{42}$ = 2, and the remainder 24.

21*g.*	2	15	10	4
				2
		31	0	8
3	7	7	15	2
		1	2	2
		39	18	0

If $\frac{1}{4}$ of a yard coſt $\frac{3}{5}$ of a pound, how much will $\frac{3}{8}$ of a yard coſt?

As $\frac{1}{4}$ ·· $\frac{3}{5}$ :: $\frac{7}{8}$ ·· $\frac{84}{40}$ = $\frac{21}{10}$ = 2*l.* 2*s.*

When 26 $\frac{1}{4}$ yards are bought for 12*l.* $\frac{7}{8}$, how much muſt be given for 31 $\frac{1}{2}$ yards?

I. By whole numbers, the yards being reduced into quarters, and the price into pence.

As 105*qr.* ·· 3090*d.* :: 126*qr.* ·· 3708*d.* = 15*l.* 9*s.*

II. By vulgar fractions.

As $\frac{105}{4}$ ·· $\frac{103}{8}$ :: $\frac{63}{2}$ ·· 15 $\frac{9}{20}$ = 15*l.* 9*s.*

III. By decimals.

As 26.25 ·· 12.875 :: 31.5 ·· 15.45 = 15*l.* 9*s.*

INVERSE PROPORTION.

WHEN the three numbers are ſtated, if the third be greater than the firſt, and yet, by the nature of the queſtion, the fourth, or anſwer, is to be leſſer than the ſecond, the proportion is inverſe.

Or, if the third be leſſer than the firſt, when the fourth muſt be greater than the ſecond, the proportion is inverſe.

But to expreſs the whole more conciſely: When more requires leſs, or leſs requires more, the proportion is inverſe. Queſtions of this kind very ſeldom occur in buſineſs.

Divide the product of the two firſt by the laſt.

EXAMPLES.

If 32 men in 12 days do a piece of work, in how many days will 48 men do the ſame quantity of work?

It is very evident that 48 men will do the work ſooner than 32 men, hence the juſtneſs of the above rule.

As 32 .. 12 :: 48 .. 8 days, for $\frac{32 \times 12}{48} = 8$ the *Anſwer*.

How long will the proviſion neceſſary for 2400 men 16 days, ſerve 960 men?

Anſwer 40 days.

COMPOUND PROPORTION.

WHEN five numbers are given to find a ſixth; or ſeven to find an eighth, &c. the proportion is called compound, requiring two or more operations, either of direct or inverſe proportion. Queſtions of this kind alſo rarely or never occur in buſineſs.

EXAMPLES.

If 400 loaves of bread ſerve 300 men 5 days, how many loaves will ſerve 720 men 20 days?

Anſwer 3840 loaves.

This is performed by two direct operations:

I. To find the loaves to ſerve 720 men for 5 days, Loaves being required, thoſe given muſt be the ſecond number.

As 300*m.* ∙∙ 400*l.* :: 720*m.* ∙∙ 960*l.*

II. To find the loaves to ſerve 720 men 20 days.

As 5*d.* ·· 960*l.* :: 20*d.* ·· 3840*l.* the *Anſwer.*

If 40 maſons in 24 days build a wall 720 yards long, and 16 feet high, in how many days will 80 maſons build a wall 840 yards long, and 12 feet high?

Anſwer 10$\frac{1}{2}$ days.

This is performed by one inverſe, and two direct operations:

I. To find in what time 80 maſons will build the firſt wall.

Inverſe, As 40*m.* ·· 24*d.* :: 80*m.* ·· 12*d.*

II. To find in what time 840 yards, 16 feet high, would be built.

Direct, As 720*y.* ·· 12*d.* :: 840*y.* ·· 14*d.*

III. To find in what time 840 yards, 12 feet high, will be built.

Direct, As 16*f.* ·· 14*d.* :: 12*f.* ·· 10$\frac{1}{2}$*d.* the *Anſwer.*

ARITHMETIC.

BOOK IV.

The Application of Practice and the Golden Rule to the other Mercantile Parts of Arithmetic.

The Rules contained in this Book, are Loſs and Gain, Fellowſhip, Tare and Tret, Alligation, Simple Intereſt, Rebate, Equation of Payments, Compound Intereſt, Annuities, Poſition, Barter and Exchange, which all depend either on Practice or the Golden Rule.

LOSS and GAIN.

CASE I.

GIVEN, the buying and ſelling prices of any quantity of goods, to find the gain or loſs *per cent.*

As the buying price is to the gain or loſs thereby, ſo is 100*l.* to the gain *per cent.*

EXAMPLES.

If muſlin be bought for 7*s.* 6*d.* per yard, and ſold for 10*s.* 6*d.* required the gain per cent.? That is, what would be the gain if 100*l.* was laid out for muſlin at 7*s.* 6*d.* per yard, and the muſlin ſold at 10*s.* 6*d.* per yard?

10 6
7 6 = 90*d.*
3 0 = 36*d.* gain by laying out 7*s.* 6*d.*

As 90*d.* ·· 36*d.* :: 100*l.* ·· the gain.

Or, as 5*d.* ·· 2*d.* :: 100*l.* ·· 40*l.*

Bought goods at 4*l.* 3*s.* 4*d.* per *cwt.* and ſold them at 5*l.* 6*s.* 8*d.* per *cwt.* required the gain per cent.?

Anſwer 28*l.*

Bought tea at 10*s.* per *lb.* and ſold it at 12*s.* per *lb.* required the gain per cent.?

Anſwer 20*l.**

When cotton is bought at 4*s.* per yard, and ſold at 3*s.* 6*d.* how much is loſt per cent.?

Anſwer 12*l.* 10*s.*

CASE

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* If the buying price be diviſible by the gain, divide 100*l.* by their proportion, the quotient is the gain per cent.

CASE II.

Given, the buying price, and the intended gain per cent. to find the ſelling price.

As 100*l.* is to 100 + the intended gain, ſo is the buying price to the ſelling price.

EXAMPLES.

If broad cloth be bought at 16*s.* per yard, how muſt it be ſold to gain 12 per cent. ?

As 100*l.* ·· 112*l.* :: 16*s.* ·· 17*s.* 11*d.* the *Anſwer.*

If ſugar be bought at 2*l.* 16*s.* per hundred weight, how muſt it be ſold per pound, to gain 20*l.* per cent. ?

Anſwer $7\frac{1}{5}$*d.*

Bought 144 gallons of rum at 8*s.* per gallon, the charges 7*l.* 4*s.* required the ſelling price to gain 18*l.* per cent. ?

SOLUTION.

	£	s.
144 gallons at 8*s.*	57	12
Charges	7	4
Paid in all	£ 64	16

As 100 ·· 118 :: 64.8 ·· 76.464, which the whole muſt be ſold for. Then $\frac{76.464}{144} = .531 =$ 10*s.* $7\frac{1}{4}$*d.* per gallon to be ſold for.

FELLOW-

FELLOWSHIP.

CASE I.

GIVEN, the ſeveral ſums entered into trade, and the gain or loſs on the whole, to find the gain or loſs for every particular ſum.

As the whole ſum entered is to the whole gain or loſs, ſo is any particular ſum to the gain or loſs proper to that ſum.

Or, divide the whole gain by the whole ſum, and multiply the quotient into every particular ſum advanced.

Or, divide the whole ſum by the gain, and divide every particular ſum by the quotient.

EXAMPLES.

Three men, A, B, and C, enter into partnerſhip, A advanced 320*l.* B 480*l.* and C 512*l.* After ſome time the whole gain was 328*l.* required each partner's ſhare of the gain, in proportion to the ſum he advanced?

	320
	480
	512
Whole ſum £	1312

Firſt Method. As 1312*l.* ·· 328*l.* :: 320*l.* ·· 80*l.* A's gain.

As 1312*l.* ·· 328*l.* :: 480*l.* ·· 120*l.* B's gain.

As 1312*l.* ·· 328*l.* :: 512*l.* ·· 128*l.* C's gain.

Proof 328*l.*

Second Method. $\frac{328}{1312} = .25$, then $320 \times .25 =$ 80*l.* A's gain; $480 \times .25 =$ 120*l.* B's gain; and $512 \times .25 =$ 128*l.* C's gain.

Third Method. $\frac{1312}{328} = 4$, then $\frac{320}{4} = 80$; $\frac{480}{4} = 120$; and $\frac{512}{4} = 128$.

D, E, and F trade in company, D advanced 272*l.* E 364*l.* and F 388*l.* they gain 432*l.* required each man's ſhare of it?

Anſwer, D gained 114*l.* 15*s.* E gained 153*l.* 11*s.* 3*d.* and F 163*l.* 13*s.* 9*d.*

A bankrupt owes to A 750*l.* to B 480*l.* to C 556*l.* to D 474*l.* and to E 740*l.* How much will each receive, the effects of the bankrupt amounting to 1000*l.*?

Anſwer, A will receive 250*l.* B 160*l.* C 185*l.* 6*s.* 8*d.* D 158*l.* and E 246*l.* 13*s.* 4*d.* therefore each creditor will receive $\frac{1}{3}$ of the ſum due to him, which is 6*s.* 8*d.* in the pound.

G, H, I, and K, have 15*l.* to pay amongſt them, in proportion to their rents. The rent of G is 500*l.* of H 780*l.*

H 780*l.* I 880*l.* and the rent of K 720*l.* how much muſt each pay?

Anſwer, G 2*l.* 12*s.* 1*d.* H 4*l.* 1*s.* 3*d.* I 4*l.* 11*s.* 8*d.* and K 3*l.* 15*s.*

A, B, and C advance each an equal ſum of money. A had his 6 months in trade, B his 8 months, and C his in trade 10 months; they gain 800*l.* required the gain of each?

Anſwer A 200*l.* B 266*l.* 13*s.* 4*d.* and C 333*l.* 6*s.* 8*d.*

C A S E II.

When both the ſums advanced, and the time of employing them, are different,

Multiply each man's part of the ſtock by the time of his continuing it in trade, then,

As the ſum of all theſe products is to the whole gain or loſs, ſo is any particular product to the gain proper to it.

Or, have recourſe to the ſecond or third methods of Case I.

Example.

A, B, and C trade in company; A had 130*l.* in trade four months; B 240*l.* ſix months; and C 300*l.* ſeven months. They gain 304*l.* 10*s.* required the gain of each?

130 × 4 = 520
240 × 6 = 1440
300 × 7 = 2100

4060

	l.	l.	l.	l.
As	4060 ··	304.5 ::	520 ··	39.
As	4060 ··	304.5 ::	1440 ··	108.
As	4060 ··	304.5 ::	2100 ··	157 10.

TARE, TRET, and CLOFF.

GROSS is the weight of both the goods and cheſts or caſks wherein they are contained.

TARE is the weight of the cheſts or caſks only, and is known by actually weighing them when empty. Generally, however, this method is not uſed, but an allowance per cwt. made, according to the practical knowledge of the merchant.

When the tare is found, and taken from the groſs, the remainder is the ſuttle.

TRET is $\frac{1}{26}$ of the ſuttle, being 4*lb.* allowed in every 104*lb.* in goods liable to waſte.

When the tret is ſubtracted from the ſuttle, the remainder is the neat, on which the charge is made, unleſs there be a farther allowance, called cloff.

CLOFF is 2*lb.* in 3*c.* ſometimes allowed for the turn of the ſcale.

EXAMPLES.

Three caſks of goods, weighing as under, required the neat, after allowing tare and tret?

	Groſs. *c.*	*qr.*	*lb.*	Tare. *qr.*	*lb.*
No. 1, …………	3	2	12	0	22
2, …………	4	1	10	0	24
3, …………	5	2	14	1	6
	13	2	8	2	24
Tare …………	·	2	24		
Suttle …………	12	3	12		
Tare …………	·	1	27 being $\frac{1}{26}$ of the ſuttle.		
Neat …………	12	1	13		

Four cheſts of goods weighing 28*c.* 3*qr.* the tare being allowed 8*lb.* per cwt. required the value of the neat at 1*l.* 3*s.* 4*d.* per *cwt.* tret, and cloff being alſo allowed?

8*lb.*	14	28	3	0	Groſs.
		2	0	6	Tare.
	26	26	2	22	Suttle.
		1	0	3	Tret.
		25	2	19	Suttle.
				17	Cloff.
		25	2	2	Neat.

By the rule of practice 25*c.* 2*qr.* 2*lb.* at 1*l.* 3*s.* 4*d.* come to 29*l.* 15*s.* 5*d.*

ALLIGATION.

CASE I.

WHEN goods of different prices are mixed, and the value of any part of the mixture or compoſition required, it is,

As the whole compoſition is to the whole value, ſo is any part of the compoſition to the value thereof.

This is called Alligation Medial.

EXAMPLES.

A farmer mixed 40 buſhels of wheat at 7*s*. 6*d*. with 60 buſhels at 6*s*. 8*d*. and 140 buſhels at 6*s*. required the price of 1 buſhel of the mixture?

40 buſhels	at 7*s*. 6*d*.	15*l*.
60	at 6*s*. 8*d*.	20*l*.
140	at 6*s*. 0*d*.	42*l*.
240		77*l*.

As 240*b*. ·· 77*l*. :: 1*b*. ·· 6*s*. 5*d*. *Anſwer*.

If 24 gallons at 8*s*. 4*d*. be mixed with 36 gallons at 6*s*. 8*d*. how much will 1 gallon of the mixture come to?

Anſwer 7*s*. 4*d*.

CASE II.

When the prices, the whole compoſition, and rate thereof, ~~are given to find the ſeveral quantities~~,

Reduce all the prices, and the rate or mean, into one denomination. Join or link a greater price than the mean to one leſſer than the mean, until all the prices are linked. If the number of prices be odd, one of them muſt have two of the prices linked to it, as in the ſecond example.

Place the difference between the mean and the ſeveral prices againſt thoſe prices to which they are reſpectively linked; for if one price exceeds the mean twice as much as the mean exceeds another, it is evident that it will only require half as many of the greater price as of the leſſer.

As the ſum of theſe differences is to the whole compoſition, ſo is any particular difference to the number at the price, oppoſite to that difference.

This is called Alligation alternate.

EXAMPLES.

How many gallons at the following prices, *viz.* at 12*s.* at 10*s.* at 8*s.* 4*d.* and at 7*s.* 6*d.* will conſtitute a mixture of 222 gallons, at 9*s.* per gallon?

108	144	18
	120	8
	100	12
	90	36
		74

As 74 ·· 222 :: 18 ·· 54 gallons at 12*s.*
As 74 ·· 222 :: 8 ·· 24 .... at 10*s.*
As 74 ·· 222 :: 12 ·· 36 .... at 8*s.* 4*d.*
As 74 ·· 222 :: 36 ·· 108 .... at 7*s.* 6*d.*

How many buſhels at the following prices, *viz.* at 6*s.* 6*d.* at 5*s.* and at 4*s.* 6*d.* will make a mixture of 294 buſhels, at 6*s.* per buſhel?

$$72 \left| \begin{array}{l} 78 \\ 60 \\ 54 \end{array} \right) \begin{array}{lcr} 12 + 18 & = & 30 \\ 6 & \ldots\ldots\ldots & 6 \\ 6 & \ldots\ldots\ldots & 6 \\ \hline & & 42 \end{array}$$

As 42 ·· 294 :: 30 ·· 210 at 6*s.* 6*d.*
As 42 ·· 294 :: 6 ·· 42 at 5*s.*
And 42 at 4*s.* 6*d.*

SIMPLE INTEREST.

WHEN money is lent, that premium or reward which is given for the loan of it, is called intereſt.

Simple intereſt is when the intereſt is received as it becomes due; or when allowed to remain in the borrower's hand without paying for it.

The ſum lent is the principal; the rate is the intereſt of 100*l.* for one year.

The amount is the ſum of the principal and intereſt.

CASE I.

Given, the principal, rate, and time, to find the intereſt.

As 100 is to the rate per cent. for one year, ſo is the principal to the intereſt for one year; which multiplied by the time produces the intereſt.

By the above rule, the principal, rate, and time, are upon the whole, multipliers, and 100, the diviſor; therefore the following more conciſe one is deduced from it.

Multiply the principal, rate, and time, and divide their product by 100, which is effected by cutting off two figures in the product.

EXAMPLES.

Required the intereſt of 750*l.* at 5 per cent. in 8 years?

METHOD I.

As 100*l.* ·· 5*l.* :: 750*l.* ·· 37.5*l.* which multiplied by 8, the product 300*l.* is the intereſt for 8 years.

METHOD II.

$\frac{750 \times 5 \times 8}{100} = 300l.$ the intereſt.

Required the intereſt of 655*l.* in 3 years, at 4 per cent.?

Anſwer 78.6 = 78*l.* 12*s.*

Required the intereſt of 235*l.* in $3\frac{1}{2}$ years, at $4\frac{1}{2}$ per cent?

$$235 \times 3.5 \times 4.5 = 37.0125l. = 37l.\ 0s.\ 3d.$$

Required the intereſt of 875*l.* 12*s.* 6*d.* in $2\frac{1}{4}$ years, at 4*l.* per cent.?

Anſwer 78*l.* 16*s.* $1\frac{1}{2}$*d.*

What is the intereſt of 1500*l.* at 4*l.* per cent. from May 12, 1779, to July 23, 1781, incluſive, being 2 years and 73 days?

Anſwer 132*l.*

What is the intereſt of 4172*l.* 13*s.* 9*d.* at $3\frac{3}{4}$ per cent. in 4 years and 96 days?

Anſwer 667*l.* 1*s.* 2*d.*

CASE II.

Given, the principal, intereſt, and rate, to find the time.

As the principal is to the intereſt, ſo is 100*l.* to its intereſt, which divide by the rate, and the quotient is the time.

Or, the following more conciſe rule is evidently deduced from it.

Multiply the principal by the rate, and thereby divide 100 times the intereſt.

EXAMPLES.

In what time will 850*l.* gain 255*l.* at 5 per cent.?

$$\frac{255 \times 100}{850 \times 5} = 6 \text{ years.}$$

In what time will 725*l.* gain 108*l.* 15*s.* at 3*l.* per cent.?

Anſwer 5 years.

CASE III.

Given, the principal, time, and intereſt, to find the rate.

Multiply the principal by the time, and thereby divide 100 times the intereſt.

An EXAMPLE.

At what rate per cent. will 600*l.* in 4 years gain 90*l.*?

Anſwer $3\frac{3}{4}$ per cent.

CASE IV.

Given, the rate, time, and intereſt, to find the principal.

Multiply the rate by the time, and thereby divide 100 times the intereſt.

EXAMPLES.

What principal, or ſum of money lent, will gain 222*l.* in 6 years, at 4*l.* per cent.?

$$\frac{222 \times 100}{6 \times 4} = 925l. \text{ principal.}$$

What principal will gain 78*l.* 16*s.* $1\frac{1}{2}$*d.* in $2\frac{1}{4}$ years, at 4 per cent.?

Anſwer 875*l.* 12*s.* 6*d.*

CASE V.

Given, the rate, time, and amount, to find the principal.

Find the amount of 100*l.* for the given time, which is done by multiplying the time by the rate, and adding the product to 100*l.* then,

As the amount of 100*l.* is to 100*l.* ſo is the given amount to the principal.

EXAMPLES.

What principal will amount to 1230*l.* in 4 years?

The amount of 100*l.* at 5*l.* per cent. in 4 years is 120*l.* *viz.* 4 times as much as in one year.

What principal will amount to 750*l.* in 2 years, at 4*l.* per cent.?

Anſwer 694*l.* 8*s.* 10*d.* ⅔

◆

REBATE, or DISCOUNT.

REBATE or diſcount is an allowance made for the preſent payment of a ſum of money due at a future period; and being of the ſame nature as the preceding caſe, is performed by the ſame rule.

EXAMPLES.

How much preſent money will pay a debt of 761*l.* 5*s.* due 1 year hence, 5*l.* per cent. being allowed for preſent payment?

The amount of 100*l.* for one year is 105*l.* at 5*l.* per cent. then,

As 105*l.* ·· 100*l.* :: 761*l.*25 ·· 725*l.* which being paid at preſent, is full ſatisfaction for 761*l.* 5*s.* due in one year, and therefore the rebate or allowance is 36*l.* 5

A legacy of 7000*l.* is to be paid in two years, but the perſon to whom it is left agrees to ſell it for ready money, allowing $4\frac{1}{2}$ per cent. rebate. How much muſt he receive at preſent?

The amount of 100*l.* in two years is 109*l.* at $4\frac{1}{2}$ per cent.

As 109 : 100 :: 7000 : 6422*l.* 0*s.* 4*d.* $\frac{44}{109}$.

How much preſent money will pay a bill of 300*l.* due in three months at 5 per cent.?

In three months 100*l.* at 5 per cent. amounts to 101.25, for the intereſt of 100*l.* for three months is 1.25, then,

As 101.25 : 100*l.* :: 300*l.* : 296*l.* 5*s.* 11*d.* the preſent money.

How much preſent money will pay a bill of 576*l.* due 42 days hence, at 5 per cent.?

Anſwer 572*l.* 14*s.* $1\frac{1}{4}$*d.*

◆ ◆ ◆

EQUATION of PAYMENTS.

WHEN ſums are due at different periods, the time for the payment of the whole is determined by equation of payments.

The common practical rule.

Multiply each ſum by the time of its becoming payable. Divide the amount of theſe products by the ſum of the payments.

The true Method.

Find the preſent value of each payment by CASE V. compute in what time the ſum of theſe preſent values will amount to the total of the payments.

An EXAMPLE.

A merchant has 200*l.* to pay in three months, and 400*l.* to pay in ſix months; he agrees to pay both together, in what time muſt that be?

METHOD I.

$200 \times 3 = 600$, and $400 \times 6 = 2400$. Then $\frac{600 + 2400}{200 + 400} = \frac{3000}{600} = 5$ months.

◆ ◆ ◆

COMPOUND INTEREST.

COMPOUND INTEREST is when the ſimple intereſt, as it becomes due, is not received, but always added to the principal, and intereſt charged for the whole.

CASE I.

Given, the principal, rate, and time, to find the amount.

METHOD I.

To the principal add the ſimple intereſt of it for one year. To this new principal add the ſimple intereſt of it for another year. Proceed in this manner to add the intereſt to the principal unto the end of the given time, or

METHOD II.

Find the amount of 1*l.* from year to year, which is done by a continual multiplication of the amount of 1*l.* for 1 year, as often as there are years given. Multiply the final amount of 1*l.* by the principal, and the product is the amount of it.

EXAMPLES.

Required the amount of 700*l.* in three years, at 5 per cent. compound intereſt?

METHOD I.

$\frac{700 \times 1 \times 5}{100}$ = 35*l.* the intereſt of 700*l.* in one year by CASE I. which added to 700*l.* the ſum 735*l.* is the principal for the ſecond year.

$\frac{735 \times 1 \times 5}{100} = 36l.75$ the intereſt of 735*l.* for one year, which added to 735*l.* the ſum 771.75*l.* is the principal for the third year.

$\frac{771.75 \times 1 \times 5}{100} = 38.5875$, the intereſt of 771.75 for 1 year, which added to 771.75, the ſum 810.3375*l.* = 810*l.* 6*s.* 9*d.* is the anſwer.

METHOD II.

The amount of 1*l.* in one year, at 5 per cent. is 1.05, therefore 1.05 × 1.05 × 1.05 × 700 = 810.3375*l.* = 810*l.* 6*s.* 9*d.* as before.

Required the amount of 600*l.* in four years, at 3 per cent. compound intereſt?

Anſwer 675*l.* 6*s.* $1\frac{1}{4}$*d.*

When the time does not conſiſt of years only, the laſt multiplier muſt only be the amount of 1*l.* for the time above the years.

Required the amount of 400*l.* in $2\frac{1}{2}$ years, at 5 per cent. compound intereſt,

1.05 × 1.05 × 1.025 × 400 = 452.025*l.* = 452*l.* 0*s.* 6*d.*

CASE II.

Given, the principal, amount, and rate, to find the time.

Divide the amount by the principal, and divide the quotient, and each ſucceeding one, by the amount of 1*l.* in one year. The number of diviſions, leſs 1, is the years. When the quotient is leſs than the amount of 1*l.* and greater than unity, find in what time 1*l.* will amount thereto.

EXAMPLES.

In what time will 700*l.* amount to 810*l.* 6*s.* 9*d.* at 5 per cent.?

I. $\frac{810.3375}{700} = 1.157625.$

II. $\frac{1.157625}{1.05} = 1.1025.$

III. $\frac{1.1025}{1.05} = 1.05.$

IV. $\frac{1.05}{1.05} = 1.$ Therefore the time is 3 years.

In what time will 2000*l.* amount to 2344*l.* 3*s.* 9$\frac{1}{4}$*d.* at 5 per cent. compound intereſt?

After four diviſions the quotient is 1.0125, to which 1*l.* will amount in $\frac{1}{4}$ of a year, and therefore the time is 3$\frac{1}{4}$ years.

CASE III.

To find the preſent worth of any ſum, due at a given time and rate, compound intereſt.

This is the reverſe to CASE I. therefore multiply the amount of 1*l.* for one year as often as there are years given, and divide the given ſum by the product. The quotient is the preſent worth.

EXAMPLES.

Required the preſent worth of 578*l.* 16*s.* 3*d.* due three years hence, at 5 per cent. compound intereſt?

$\frac{578.8125}{1.05 \times 1.05 \times 1.05}$ = 500*l.* the preſent worth; or which is the ſame, it is the principal, which in three years would amount to 578*l.* 16*s.* 3*d.* at compound intereſt.

Required the preſent worth of 1000*l.* due 4½ years hence, at 3 per cent. compound intereſt?

Anſwer 875*l.* 7*s.* 1½*d.*

ANNUITIES.

AN Annuity is a ſum of money due yearly.

ANNUITIES

AT

SIMPLE INTEREST.

To find the amount of an annuity in a given time at a given rate.

An annuity only becomes due at the end of the firſt year, and therefore intereſt does not commence until the end of two years.

Multiply the progreſſive amount of 1*l.* for the given time by the annuity.

An EXAMPLE.

Required the amount of 60*l.* annuity, forborn four years, at 5 per cent. ſimple intereſt?

The amount of 1*l.* in 3 years 1.15
.... 2 years 1.1
.... 1 year 1.05
The annuity 1

4.3

$4.3 \times 60 = 258l.$ the *Anſwer*.

If the amount be given, it is eaſy to ſee in what manner the annuity may be found, *viz.* by dividing the amount by the progreſſive amount of 1*l.* for the time.

ANNUITIES

AT

COMPOUND INTEREST.

CASE I.

To find the amount of an annuity forborn any number of years, compound intereſt.

Find the amount of 1*l.* annuity for the given time, by finding the amount from year to year, and multiply by the annuity.

An EXAMPLE.

Required the amount of 40*l.* per annum, forborn five years, at 5 per cent. compound intereſt?

1*l.* annuity.

1 × 1.05, + 1 = 2.05 amount at the end of two years.

2.05 × 1.05, + 1 = 3.1525 amount in three years.

3.1525 × 1.05, + 1 = 4.310125 amount in four years.

4.310125 × 1.05, + 1 = 5.52563125 amount in five years.

5.52563125 × 40 = 221.02525 = 221*l.* 0*s.* 6*d.* the amount.

From the preceding operation the manner of conſtructing tables of the amount of 1*l.* at any rate that can be given, and for any number of years, is evident. When the time is many years, theſe tables are extremely uſeful; but as thoſe who have much occaſion to compute annuities are furniſhed with books containing all rates of intereſt for a great number of years, I have only given a table of the amount of 1*l.* at 5 per cent. as far as 21 years, by which, however, any other table of the kind is explained.

A TABLE,

Containing the Amount of 1*l. Annuity, at* 5 *per cent. as far as Twenty-one Years.*

Years.	Amount.	Years.	Amount.	Years.	Amount.
1	1.000000	8	9.549109	15	21.578564
2	2.05	9	11.026564	16	23.657491
3	3.1525	10	12.577892	17	25.840366
4	4.310125	11	14.206787	18	28.132384
5	5.525631	12	15.917126	19	30.539003
6	6.801913	13	17.712982	20	33.065954
7	8.142008	14	19.598632	21	35.719252

EXAMPLES *by the* TABLE.

Required the amount of an annuity of 80*l.* forborn 16 years, at 5 per cent. compound intereſt?

$23.657491 \times 80 = 1892.59928 = 1892l.\ 11s.\ 11\frac{3}{4}d.$

Required the amount of an annuity of 120*l.* 10*s.* forborn 10 years, at 5 per cent. compound intereſt?

Anſwer 1515*l.* 12*s.* 8½*d.*

CASE II.

To find the preſent worth of an annuity to continue a given number of years, allowing compound intereſt for the purchaſe money.

Divide the annuity by the amount of 1*l.* for one year, the quotient will be the preſent worth of the firſt year's annuity.

Divide the preſent worth of the firſt year's annuity by the amount of 1*l.* for one year, the quotient is the preſent worth of the ſecond year's annuity.

From the preſent worth of the ſecond year's annuity find the preſent worth of the third year's, &c. The ſum of theſe is the preſent worth of the annuity for the given time.

An EXAMPLE.

Required the preſent worth of an annuity of 60*l.* to continue five years.

$$\left.\begin{array}{rl} \frac{60}{1.05} & = 57.1428 \\ \frac{57.1428}{1.05} & = 54.4218 \\ \frac{54.4218}{1.05} & = 51.8302 \\ \frac{51.8302}{1.05} & = 49.3621 \\ \frac{49.3621}{1.05} & = 47.0116 \end{array}\right\} \begin{array}{c}\text{Preſent}\\ \text{worth}\\ \text{of the}\end{array} \left\{\begin{array}{c} 1\text{ſt} \\ 2\text{d} \\ 3\text{d} \\ 4\text{th} \\ 5\text{th} \end{array}\right\} \begin{array}{c}\text{Year's}\\ \text{annuity.}\end{array}$$

Anſwer £ 259.7685 = 259*l.* 15*s.* 4½*d.*

But the preſent worth of any annuity is much more readily found by tables that contain the preſent worth of 1*l.* annuity for any number of years at any intereſt, computed by the preceding method.

A TABLE

Containing the preſent worth of 1*l. Annuity as far as* 12 *years, at* 5 *per cent. compound intereſt.*

Yrs.	Preſent worth.	Yrs.	Preſent worth.	Yrs.	Preſent worth.
1	.952381	5	4.329476	9	7.107821
2	1.859410	6	5.075692	10	7.721735
3	2.723248	7	5.786373	11	8.306414
4	3.545950	8	6.463213	12	8.863251

An EXAMPLE *by the* TABLE.

Required the preſent worth of an annuity of 200*l.* to continue eight years?

6.463213 × 200 = 1292.6426 = 1292*l.* 12*s.* 10¼*d.*

SINGLE

SINGLE POSITION.

POSITION is ſometimes called the Rule of Falſe, becauſe falſe numbers are aſſumed to diſcover the true ones by.

Single poſition only requires one ſuppoſition; double poſition requires two ſuppoſitions. *See* BOOK V.

In ſingle poſition, as the ſum of the ſuppoſed numbers is to the ſum of the real ones, ſo is any ſuppoſed number to the correſponding real one.

An EXAMPLE.

A, B, C, and D have 1330*l.* to receive; B to have twice as much as A, C thrice as much as B, and D five times as much as A. How much will each receive?

	£
Suppoſe A receives	10
Then, by the queſtion, B will receive	20
C	60
D	50
	140

As 140 ·· 1330 :: 10 ·· 95*l.* the real ſum for A, conſequently B will receive 190*l.* C 570*l.* and D 475*l.*

BARTER.

BARTER.

BY this rule goods of one ſort are exchanged for goods of another.

Multiply the given quantity by the price thereof, and divide the product by the price of the quantity required.

EXAMPLES.

How many gallons of wine, at 6*s.* 4*d.* per gallon, muſt be given for 342 yards of muſlin, at 10*s.* 8*d.* per yard?

$$\frac{342 \times 128}{76} = 576 \text{ yards.}$$

How many yards of ſhalloon, at 1*s.* 9*d.* per yard, muſt be given for 73*lb.* of tea, at 7*s.* per *lb.*?

$$\frac{73 \times 84}{21} = 73 \times 4 = 292 \text{ yards.}$$

A and B barter, A gives B 640 gallons of wine, at 6*s.* 8*d.* per gallon, for which B gives A 512 yards of ſilk. Required the value of one yard of ſilk?

$$\frac{640 \times 80}{512} = 100d. = 8s.\,4d. \text{ per yard.}$$

C and D barter, C gives D 532 yards of linen, at 1*s.* 6*d* per yard, for which D gives him 30*l.* in caſh, and the remainder in wine at 6*s.* per gallon. How many gallons muſt C receive?

Anſwer 33 gallons.

EXCHANGE.

BY this rule the money of one country is exchanged for the money of another.

Par of exchange is when the money received in one country is of the ſame abſolute value as the money delivered in another.

The courſe of exchange is ſometimes above par, ſometimes under it, but very ſeldom equal to par.

The par of exchange between London and Holland, Flanders, *&c.* is 33*s.* 4*d.* Flemiſh, for 1*l.* ſterling.

With France, the par is 4*s.* 6*d.* ſterling for 1 French crown.

The par with Spain and Italy is 4*s.* 6*d.* for one dollar, or piece of eight.

The par with Portugal is 6*s.* 8½*d.* for 1 milrea, = 1000 rees.

Exchange is performed by the rule of practice.

EXAMPLES.

How many Flemiſh pounds are equal to 720*l.* ſterling, the courſe of exchange being 34*s.* 10*d.* or 1*l.* 14*s.* 10*d.* Flemiſh* for 1*l.* ſterling?

1*l.* Flemiſh		720
10*s.*	2	360
4*s.*	5	144
10*d.*	12	30
		1254 Flemiſh pounds.

Or, 34*s.* 10*d.* = 418*d.* and 720 × 418 = 300960 Flemiſh pence, which being divided by 12 and 20, the laſt quotient is 1254*l.* Flemiſh, as before.

How many Flemiſh pounds are equal to 472*l.* 16*s.* ſterling, the courſe being 35*s.* 4*d.*?

Anſwer 835*l.* 5*s.* 7⅕*d.*

---

* One pound Flemiſh contains 20 Flemiſh ſhillings, each ſhilling 12 Flemiſh pence, therefore 1 pound Flemiſh is equal to 12 ſhillings ſterling, and 1 Flemiſh ſhilling to 7 pence one-fifth, at par.

How much ſterling is equal to 1072*l* Flemiſh, the courſe being 33*s*. 6*d*. ?

$$\frac{1072 \times 20 \times 12}{33s.\ 6d. = 402} = 640l.\ \text{ſterling.}$$

How much ſterling is equal to 576*l*. Flemiſh, at par?

Anſwer 384*l*.

How much do 480 French crowns come to, the courſe of exchange being 4*s*. 8*d*. per crown?

Anſwer 112*l*.

How many French crowns, at par, are equal to 97*l*. 4*s*.?

Anſwer 432 crowns.

How much do 745 milreas come to, the courſe being 6*s*. 8*d*. ?

Anſwer 248*l*. 6*s*. 8*d*.

ARITHMETIC.

BOOK V.

Rules not applicable to Mercantile Purpoſes, but uſeful in the Mathematics, *viz.* Involution, Evolution, Progreſſion, and Double Poſition.

INVOLUTION;

OR,

RAISING POWERS.

WHEN a number is multiplied by itſelf, it is ſquared, or involved into the ſecond power.—For inſtance, 5 × 5 = 25, is the ſquare or ſecond power of 5. The number given to be ſquared, in this inſtance 5, is called the root.

If the ſquare be multiplied by the root, the product is the cube or third power, thus $5 \times 5 \times 5$, or $25 \times 5 = 125$, is the cube of 5.

When the cube is multiplied by the root, the product is the biquadrate, or ſourth power; for inſtance, $5 \times 5 \times 5 \times 5$, or $125 \times 5 = 625$, is the biquadrate of 5.

From theſe definitions, it is eaſy to comprehend, that any power, however high, is formed or generated, by continually multiplying by the root. But powers are moſt readily raiſed from each other thus, a ſquare multiplied by a ſquare produces a biquadrate; a cube by a cube, or a biquadrate by a ſquare, produces the ſixth power, &c.

EXAMPLES.

Involve 24 into the ſurſolid, or fifth power.

$24 \times 24 \times 24 \times 24 \times 24 = 7962624$, the ſurſolid.

Or $\overline{24}|^{3} \times \overline{24}|^{2} = 13824 \times 576 = 7962624$, being the ſquare of the cube.

The expreſſion $\overline{24}|^{3}$ repreſents the cube of 24. The 3 is called the exponent. The exponent of the ſquare is 2; of the biquadrate, 4; of the fifth power, 5, &c.

Involve 16 into the ſeventh power.

$\overline{16}|^{3} \times \overline{16}|^{2} \times \overline{16}|^{2} = \overline{16}|^{7} = 268435456$.

If there be an integral proportion between two roots, and ſome power of the leſſer root known, the ſame power of the greater root is readily found by multiplying the known power by the ſame power of the proportion between the two roots.

EXAMPLES.

The cube of 26 is 17576, required thence the cube of 52?

The proportion between the roots is 2, whoſe cube is 8, by which 17576 being multiplied, the product is 140608.

The ſquare of 78 is 6084, required thence the ſquare of 234?

The proportion of the roots is 3, whoſe ſquare 9, multiplied into 6084, produces 54756.

A TABLE

Of Roots and Powers.

Root -	1	2	3	4	5	6	7	8	9
Square -	1	4	9	16	25	36	49	64	81
Cube -	1	8	27	64	125	216	343	512	729
Biquadrate	1	16	81	256	625	1296	2401	4096	6561
Surſid	1	32	243	1024	3125	7776	16807	32768	59049

EVOLUTION.

BY evolution the root of any number is known. For inſtance, the ſquare root of 36 is 6; the cube root of 1728 is 12; the biquadrate root of 256 is 4, &c.

EXTRACTION

OF THE

SQUARE ROOT.

IF the ſquare conſiſts of an even number of figures, the firſt period contains two of them; but if the ſquare conſiſts of an odd number of figures, the firſt figure is the firſt period.

The root of the neareſt ſquare to the firſt period is the firſt figure in the root. Subtract that neareſt ſquare from the firſt period, and to the remainder bring two figures to conſtitute a minuend.

Double the root for a diviſor, and, as in diviſion, enquire how often the diviſor is contained in the minuend, neglecting the laſt figure. Place the quotient figure in the root, and alſo join it to the right of the diviſor, after which multiply by it, and ſubtract the product from the minuend.

To the remainder bring two figures to make a new minuend. Double the root for a new diviſor, and enquire as before how often it is contained in the new minuend, neglecting always the laſt figure of the minuend.

Proceed in this manner until all the figures of the given ſquare are uſed, *viz.* to every remainder bring two figures, and always double all the root for a new diviſor.

EXAMPLES.

Extract the ſquare root of 4624. That is, find a number, which, multiplied by itſelf ſhall produce 4624.

```
          4624 ( 68 Root.
          36
         ----
    128 ) 1024
          1024
         ----
          ....
```

68 × 68 = 4624, Proof.

Extract the ſquare root of 138384.

```
        138384 ( 372 Root.
        9
       ---
   67 ) 483
        469
       -----
  742 )  1484
         1484
       ------
          ....
```

Required the ſquare root of 556516.

Anſwer 746.

Required the ſquare root of 28987456.

Anſwer 5384.

Required the ſquare root of 81796.

Anſwer 286.

As 81796 conſiſts of an odd number of figures, *viz.* five, the firſt period contains only one figure, that is 8, but two figures are brought into the minuend together, as in the other examples.

Required the ſquare root of 2729104.

Anſwer 1652.

When the diviſor is not contained in the minuend, put a cypher into the diviſor, another into the root, and bring two figures more to the minuend.

Extract the ſquare root of 29224836.

Anſwer 5406.

Extract the ſquare root of 1849516036.

Anſwer 43006.

If there be decimals, cut off half as many in the root as there are in the given ſquare.

Extract the ſquare root of 388627.56.

Anſwer 623.4.

Extract the ſquare root of 85.3776.

Anſwer 9.34.

When there is a remainder, whether there be decimals or not, the given number is called a ſurd. If more figures be neceſſary in the root, join two cyphers to the remainder, and to every ſucceeding remainder, until there are a ſufficient number of decimals in the quotient. Two cyphers produce one decimal in the root.

Extract the ſquare root of 8532, and continue the extraction until there are three decimals in the root.

Anſwer 92.368.

Extract the ſquare root of 7462.6324, and continue the extraction until there are four decimals in the root.

Anſwer 86.3865.

Extract the ſquare root of 282, and have four decimals in the root.

Anſwer 16.7928.

Extract the ſquare root of .5329.

Anſwer .73.

Extract the ſquare root of .8642.

Anſwer .92.

If there be an odd number of decimals, join a cypher to them before any part of the operation is begun.

Extract the ſquare root of 472.825.

Anſwer 21.74.

Extract the ſquare root of .726.

Anſwer .85.

Extract the ſquare root of .123, until there are four decimals in the root.

Anſwer .3507.

When the root of a fraction has to be extracted, find the root of the numerator, and the root of the denominator, if they be both perfect ſquares; but if they are not, reduce the fraction into a decimal, and extract the ſquare root thereof.

Required the ſquare root of $\frac{9}{121}$.

Anſwer $\frac{3}{11}$.

Extract the ſquare root of $\frac{225}{1156}$.

Anſwer $\frac{15}{34}$.

Extract the ſquare root of $\frac{7}{8}$.

Anſwer .9354.

EXTRAC-

EXTRACTION
OF THE
CUBE ROOT.

DIVIDE the number of figures of which the given cube conſiſts by 3, not including the decimals, if any. When nothing remains, the firſt three figures conſtitute the firſt period; if the remainder be 2, the firſt period muſt conſiſt of the two firſt figures; and if 1 remains, the firſt figure is the firſt period.

The root of the neareſt cube to the firſt period is the firſt figure of the root. Subtract that neareſt cube from the firſt period, and to the remainder bring three figures to conſtitute a minuend.

Multiply the ſquare of the root always by 300 for a diviſor, and find how often the diviſor is contained in the minuend. Place the quotient figure in the root, and multiply the diviſor by it. Call this product A.

Square the laſt figure of the root; multiply that ſquare by the preceding part of the root, and the product always by 30. Call this product B.

To the product A add the product B, and the cube of the laſt figure of the root. Subtract the ſum from the minuend, and bring three figures to the remainder to make a new minuend.

Multiply the ſquare of the root by 300 for a new diviſor, and with this diviſor and minuend, and with all the following, proceed as with the former.

Every minuend is found by bringing three figures to the remainder; and every diviſor, by multiplying the ſquare of all that part of the root which is found, by 300.

EXAMPLES.

Extract the cube root of 584277056.

The firſt period of this cube is 584, for when the number of figures of which the cube conſiſts, *viz.* 9, is divided by 3, there is no remainder. The neareſt cube to 584 is 512, whoſe root is 8. *See the* TABLE *of* POWERS, *page* 160.

584277056 (836 cube root.

512

19200) 72277 minuend.

57600 A. The diviſor × 3 = 57600 = A.

2160 B. 3 × 3 × 8 × 30 = 2160 = B.

27 cube of 3.

59787

2066700) 12490056 minuend.

12400200 A. Diviſor × 3 = 14200200 = A.

89640 B. 6 × 6 × 83 × 30 = 89640 = B.

216 cube of 6.

12490056

....

8 × 8 × 300 = 19200, the firſt diviſor.

83 × 83 × 300 = 2066700, the ſecond diviſor.

Proof 836 × 836 × 836 = 584277056

Extract the cube root of 157464.

Anſwer 54.

Extract the cube root of 83797669376.

Anſwer 4376.

When the diviſor is not contained in the minuend, put two cyphers into the diviſor, and bring three figures to the minuend.

Extract the cube root of 32891033664

Anſwer 3204.

Extract the cube root of 729583355533824.

Anſwer 90024.

If there be decimals, cut off one-third of their number in the root.

Extract the cube root of 13.144256.

Anſwer 2.36.

When the number of the given decimals is not diviſible by 3, add a cypher or two to them before the operation.

Extract the cube root of 938764.2746.

Anſwer 97.91

Extract the cube root of $\frac{2744}{4913}$.

Anſwer $\frac{14}{17}$.

Required the cube root of $\frac{5}{12}$.

Anſwer .746.

EXTRACTION

IN

GENERAL.

A GENERAL Rule for extracting the root of any power.

If the number of figures, not including decimals, if any, be diviſible by the exponent, the number of figures in the firſt period, as well as in the others, will be expreſſed by the exponent. When there is a remainder, it ſhews the number of figures in the firſt period; the exponent always expreſſes the number in the other periods.

Subtract the neareſt biquadrate, ſurſolid, &c. according to the given power, from the firſt period, and bring the firſt figure of the ſecond period to the remainder, to make a minuend.

Involve the root until the exponent is within 1 of the given power, and multiply by the exponent of the given power for a diviſor, and find another figure to be placed in the root.

Involve that part of the root which is found, into the given power, and ſubtract the reſult from the two firſt periods, and bring the firſt figure of the third period to the remainder.

Find a diviſor by involving the root, and multiplying by the exponent of the power, as before. In the ſame manner every future diviſor is found.

If the root when involved is greater than the periods from whence it is to be taken, the laſt figure of the root muſt be diminiſhed.

EXAMPLES.

Extract the biquadrate root of 856446597136.

856446597136 (962 *Anſwer.*
6561

2916) 20034

84934656 ſubtrahend.

3538944) 7100037

$9^3 \times 4 = 2916$, the firſt diviſor.

$96^3 \times 4 = 3538944$, ſecond diviſor.

Proof $962 \times 962 \times 962 \times 962 = 856446597136$.

The biquadrate root may alſo be found by extracting the ſquare root twice. The ſquare root of 8564465597136 is 925444, and the ſquare root of 925444 is 962, the ſame as before.

Extract the ſurſolid root of 1561149281279148032.

Anſwer 4352.

Extract the root of the ſeventh power, 340.4825447.

Anſwer 2.3

Add cyphers to the decimals to make them proper periods, when they are not ſo.

Extract the root of the eighth power, 9462.43425.

Anſwer 3.1

◆ ◆ ◆

The root of the eighth power may be alſo found by extracting the ſquare root thrice; for it is the ſquare of the biquadrate, which is the ſquare of the ſquare.

The root of the ninth power may be found by extracting the cube root twice, for it is the cube of the cube.

The root of the tenth power may be found by extracting the ſquare and ſurſolid roots.

In general, if the exponent conſiſts of two or more components, theſe ſhew the extractions whereby the root may be found. The components of the twelfth power, for inſtance, are 3, 2, and 2, for 3 × 2 × 2 = 12, and conſequently the root of the twelfth power may be found by extracting the cube root, and twice the ſquare root.

When a number has a fractional exponent, involve according to the numerator, and evolve according to the denominator.

EXAMPLES.

Required the root of $16^{\frac{3}{2}}$

The cube of 16 is 4096, whoſe ſquare root is 64.

Required the root of $7^{\frac{5}{2}}$

Anſwer 129.64.

ARITHMETIC PROGRESSION.

WHEN any rank or ſeries of numbers increaſe by a common addend, or decreaſe by a common ſubtrahend, theſe numbers are in arithmetic progreſſion, or proportion.

For inſtance, 3, 5, 7, 9, 11, 13, and 15, are in arithmetic proportion, the common difference being 2.

Again, 43, 38, 33, 28, 23, 18, 13, 8 and 3 are in arithmetic proportion decreaſing, the common difference being 5.

Every ſeries conſiſts of the five following parts:

The leaſt term,
The common difference,
The number of terms,
The greateſt term, and
The ſum of the ſeries

Any three of theſe being given, the others may be very readily found.* The two following caſes are the principal ones.

CASE I.

Given, the leaſt term, the common difference, and number of terms, to find the greateſt term.

Multiply the number of terms, leſs 1, by the common difference, and add the leaſt term to the product.

EXAMPLES.

The leaſt term of a ſeries in arithmetic progreſſion being 5, the common increaſe 4, and number of terms 24, required the greateſt term?

23	Number of terms, leſs 1.
4	Common difference.
92	Product.
5	Leaſt term.
97	Greateſt term.

---

* Algebra, *page* 207.

The maſter of an academy begins with 5 pupils; every week 3 enter; how many will he have in 20 weeks?

Anſwer 65.

CASE II.

Given, as before, to find the ſum of the ſeries.

Multiply the number of terms, leſs 1, by the common increaſe; add twice the leaſt term, and multiply half the ſum, by the number of terms.

An EXAMPLE.

Required the ſum of all the whole numbers that are under 101.?

The number of terms leſs 1, is	99
Common increaſe, multiply	1
	99
Twice leaſt term	2
	101
Half the number of terms	50
Anſwer	5050

GEOMETRIC PROGRESSION.

WHEN any rank or ſeries of numbers increaſe by a common multiplier, or decreaſe by a common diviſor, theſe numbers are in geometric progreſſion, or proportion.

For inſtance, 4, 12, 36, 108, 324, 972, and 2916 are in geometric proportion, the multiplier or ratio 3.

Again 1024, 256, 64, 16, 4, 1, are in geometric progreſſion, the common diviſor 4.

CASE I.

Given, the leaſt term, ratio, and number of terms, to find the greateſt term.

Involve the ratio until the exponent is 1 leſs than the number of terms, and multiply by the firſt term.

EXAMPLES.

The leaſt term of a ſeries in geometric progreſſion being 6, the ratio 2, and number of terms 21, required the greateſt term?

2 × 2 × 2 × 2 × 2 = 32, the fifth power.
32 × 32 = 1024, the tenth power.
1024 × 1024 = 1048576, the twentieth power.
1048576 × 6 = 6291456, greateſt term.

The leaſt term of a geometric ſeries being 8, the ratio 8, and number of terms 8. Required the greateſt term?

Anſwer 16777216.

CASE II.

Given, the leaſt term, ratio, and number of terms, to find the ſum of the ſeries.

Find the greateſt term by the preceding caſe; multiply it by the ratio; ſubtract the leaſt term, and divide by the ratio, leſs 1.

EXAMPLES.

The leaſt term of a geometric ſeries being 5, the ratio 4, and number of terms 15, required the greateſt term, and ſum of the ſeries?

4 × 4 × 4 × 4 = 256, the fourth power.
256 × 256 × 256 = 16777216, the twelfth power.
16777216 × 16 = 268435456, the nineteenth power.
268435456 × 5 = 1342177280, the greateſt term.

	1342177280	Greateſt term.
	4	Ratio.
	5368709120	
	5	Leaſt term.
Ratio — 1 = 3)	5368709115	
	1789569705	Sum of the ſeries.

Two young gentlemen, A and B, ſpeaking of the intended marriage of the former, A ſaid, give me 10,000*l.* and I will give you two-pence to-day, ſix-pence to-morrow, eighteen-pence the third day, and increaſe in that manner every day until I am married. B agreed, and in 20 days A married. Which of them had the advantage?

3 × 3 × 3 × 3 = 81, the 4th power of the ratio.
81 × 81 = 6561, the 8th power.
6561 × 6561 × 27 = 1162261467, the 19th power.
1162261467 × 2 = 2324522934, greateſt term, or the pence paid on the laſt day.

To find the ſum of all the payments, *viz.* the ſum of the ſeries.

2324522934
3 Ratio.

6973568802
2

Ratio — 1 = 2)6973568800

3486784400 Pence, =

14528268*l.* 6*s.* 8*d.* therefore B had very much the advantage.

CASE III.

Given, the greateſt term, in a deſcending or decreaſing ſeries, the diviſor, and number of terms, to find the leaſt term.

Involve the diviſor until the exponent is 1 leſs than the number of terms, and thereby divide the greateſt term.

EXAMPLES.

The greateſt term of a deſcending geometric ſeries being 6144, the common diviſor 2, and number of terms 12, required the leaſt term, and ſum of the ſeries?

$2 \times 2 \times 2 \times 2 \times 2 = 32$, the fifth power of the diviſor.

$32 \times 32 \times 2 = 2048$, the 11th power.
$\frac{6144}{2048} = 3$, the leaſt term.

When the leaſt term is found, the ſum of the ſeries is known by the preceding caſe, and is in this example 12285.

The preſent worth of annuities may be found by this caſe, for the ſeveral annual values are a ſeries of numbers in geometric progreſſion, the annuity divided by the amount of 1*l.* in 1 year being the greateſt term; the amount of 1*l.* for 1 year, the common diviſor; and the number of years being the number of terms.

Required the preſent worth of an annuity of 400*l.* to continue 4 years, at 5 per cent. compound intereſt?

Anſwer 1418*l.* 7*s.* 7$\frac{1}{4}$*d.*

If the terms of a geometric ſeries be infinite, the laſt term will be nothing, and therefore the ſum of the ſeries will be found by multiplying the greateſt term by the ratio, and dividing by the ratio, leſs 1.

Required the ſum of the infinite ſeries, 20, $\frac{20}{3}$, $\frac{20}{3 \times 3}$, $\frac{20}{3 \times 3 \times 3}$, $\frac{20}{3 \times 3 \times 3 \times 3}$, &c. or 20, $\frac{20}{3}$, $\frac{20}{9}$, $\frac{20}{27}$, $\frac{20}{81}$, &c.?

Anſwer 30, for $\frac{20 \times 3}{3 - 1} = \frac{60}{2} = 30$.

Required the ſum of the infinite ſeries $\frac{100}{1.05}$, $\frac{100}{1.05 \times 1.05}$, $\frac{102}{1.05 \times 1.05 \times 1.05}$, &c.

Anſwer 2000*l.* for $\frac{100}{1.05} \times 1.05 \div .05 = \frac{100}{.05} = 2000$*l.*

By the above example the preſent worth of any yearly income, to continue for ever, may readily be found.

* * *

DOUBLE POSITION.

BY this rule ſome of the eaſieſt queſtions of algebra may be anſwered, but neither ſo conciſely nor ſo elegantly as by that moſt excellent ſcience.

It is called double poſition, becauſe it requires two ſuppoſitions.

Multiply the firſt ſuppoſed number by the ſecond error, and the ſecond ſuppoſed number by the firſt error.

If the two errors are alike, that is, both too great, or both too little, ſubtract the two products, and divide by the difference of the two errors.

But if the errors are unlike, that is, the one too great, and the other too little, add the products, and add the errors.

EXAMPLES.

B has 12 more than twice as many ſheep as A; C has 18 more than thrice as many as B. They have among them 705. How many belong to each?

FIRST SUPPOSITION.

Suppoſe A	30
Then B 30 × 2 + 12	72
C 72 × 3 + 18	234
	336

705
336
369 Firſt error.

SECOND SUPPOSITION.

Suppoſe A	60
Then B	132
And C	414
	606

$$\begin{array}{r} 705 \\ 606 \\ \hline 99 \end{array} \text{ Second error.}$$

$30 \times 99 = 2970$, and $60 \times 369 = 22140$.

$\frac{22140 - 2970}{368 - 99} = \frac{19170}{270} = 71$ belonging to A; conſequently B 154, and C 480.

A lady ſaid, that 12 years hence ſhe will be half as old as her father, that ſhe is 20 years younger than her mother, and that the ſum of their three ages at preſent is 104. Required the age of each?

Anſwer, the age of the young lady is 18 years, of her mother 38, ~~and of~~ her father 48.

E has 20 guineas more than D, who has 4 more than C, and C has half as many as E. Required the number in the poſſeſſion of each?

Anſwer C has 24 guineas, D 28, and E 48.

Required two numbers, twice their ſum 248, and thrice their difference 54?

Anſwer the greater 71, and the leſſer 53.

The ſum of a ſeries of 7 terms in arithmetic progreſſion being 105, and the firſt term twice the common difference, required each term?

Anſwer 6, 9, 12, 15, 18, 21, and 24.

THE END OF ARITHMETIC.

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A
GEOGRAPHICAL
TABLE.

BRITISH MUSEUM

THE direct diſtance and bearing from London of all the principal towns, ports, capes, ſtraits, and rivers, and the probable time of ſailing from London to the ſeveral ports.

E ſtands for Europe; A for Aſia; Af. Africa; N. A. North America; and S. A. South America.

In the bearing column, N. ſtands for North; S. for South; E. Eaſt; and W. Weſt; and conſequently N. E. b E. ſtand for North-eaſt by Eaſt; E. S. E. for Eaſt-South-Eaſt, &c.

An inland place is known by having a line drawn through the laſt column, thus Bagdat, Berlin, &c. are inland towns.

To places marked *, the courſe is direct, either from London, or ſome other Engliſh port, for inſtance, the courſe from England to Boſton is direct.

I. ſtands for iſland.

Diſtance and Bearing from London, &c,

	Diſtance in Engliſh miles.	Bearing.	Time of ſailing in days.
A			
Aberdeen, Scotland, E.	411	N.	2
Abo, Sweden, E.	1028	N. E. b E.	10
Acapulco. Pacific Ocean, S. A.	6220	W.S.W.	164
Alexandria, Egypt, A.	2129	S.E.	42
Algiers, Barbary, Af.	1045	S.b E.	24
Alicant, Spain, E.	880	S.	24
Amazon river,* Atlantic Ocean, S. A.	4622	S.W.bW.	50
Amſterdam.* Holland. E.	318	E.bN.	3
Antigua,* I. W. Indies	4140	S.W.bW.	45
Aſcenſion, I. Atlantic Ocean, Af.	1645	S.bW.	50
Aſtracan, Siberia, A.	2430	E.bS.	—
B			
Bagdat, Turkey, A.	2530	S.E.bE.	—
Barbadoes,* I. W. Indies	4300	S.W.bS.	48
Barcelona, Spain, E.	703	S.bE.	28
Bencoolen, Sumatra I. East Indies	4262	S.E.bE.	144
Bergen,* Norway, E.	629	N.N.E.	8
Berlin, Pruſſia, E.	572	E.	—
Bermuda,* I. Atlantic Ocean, N. A.	3535	W.S.W.	40
Bern, Switzerland, E.	454	S.E.	—

Diſtance and Bearing from London, &c.

	Diſtance in Engliſh miles.	Bearing.	Time of ſailing in days.
Berwick, England, E.	304	N.bW.	6
Bilboa,* Bay of Biſcay, Spain	507	S.bE.	7
Bremen, Germany, E.	382	E.N.E.	5
Breſt, France, E.	286	S.W.	6
Briſtol, England, E.	111	W.	8
Bombay, Eaſt Indies, A.	4590	S.E.bE.	140
Boſton, * New England, N. A.	3390	W.bS.	38
C			
Cadiz, Spain, E.	1111	S.S.W.	19
Calcutta, Eaſt Indies, A.	5100	S.E.bE.	134
Candia I. Mediterranean, E.	1356	S.E.	37
Canton, China, A.	6409	E.S.E.	170
Cape Bona, Barbary, A.	1104	S.S.E.	30
——Bonaviſta,*Newfoundland, N. A.	2200	W.	26
—— Clear, Ireland, E.	423	W.	8
—— Coromin, E. Indies	5352	S.E.bE.	136
—— Finiſterre,* Spain, E.	945	S.W.bS.	12
—— Florida,* Florida, N. A.	4786	W.S.W.	54
—— Good Hope, Af.	6068	S.b E	80
—— Horn, S. A.	8900	S.W.bS.	104
—— Matapan, Mediterranean, E.	1527	S.E.	37
—— May, Atlantic Ocean, N. A.	3670	W.bS.	42

Diſtance and Bearing from London, &c.

	Diſtance in Engliſh miles.	Bearing.	Time of ſailing in days.
Cape Naſſau,* Terra Firma, S. A.	4591	S.W.	52
—— Palmas, Guinea, Af.	3500	S.bW.	44
—— St. Auguſtine,* S. A.	4678	S.S.W.	58
—— St. Lucas, California, S. A.	6361	W.S.W.	170
—— St. Sebaſtian, Madagaſcar I. Af.	6037	S.E.bE.	101
—— St. Vincent, Portugal, E.	1114	S.S.W.	15
—— Southampton, Hudſon Bay, N. A.	3422	W.bN.	45
—— Verd, Negroland, Af.	2737	S.S.W.	30
—— Victory, Pacific Ocean, S. A.	8753	S.W.b.	109
Charles Town,* South Carolina, N. A.	4224	W.bS.	47
Cherry I. North Sea, E.	1690	N.N.E.	18
Cochin, Eaſt Indies, A.	5226	S.E.bE.	133
Cologne, Germany, E.	250	E.bS.	—
Columbo, Ceylon I. Eaſt Indies	5460	S.E.bE.	138
Conſtantinople, Turkey, E.	1584	E.S.E.	47
Copenhagen, Denmark, E.	594	N.E.bN.	9
Cork, Ireland, E.	382	W.	8
Corſica I. Mediterranean Sea, E.	780	S.E.bS.	29
Cracow, Poland, E.	857	E.bS.	—
Cyprus I. Mediterranean Sea, E.	2000	S.E.bE.	35

Diſtance and Bearing from London, &c.

	Diſtance in Engliſh miles.	Bearing.	Time of ſailing in days.
D			
Dantzick, Poland, E.	776	E.bN.	13
Delly, Eaſt Indies, A.	2619	E.S.E.	—
Dierbeck, Turkey, A.	2278	E.S.E.	—
Dominica I. W. Indies	4210	S.W.bW.	47
Dreſden, Germany, E.	568	E.	—
Dublin, Ireland, E.	332	W.N.W.	9
E			
Edinburgh, Scotland, E.	331	N.N.W.	7
F			
Falkland Iſ.* S. A.	7980	S.S.W.	88
Ferro I.* Atlantic Ocean, Af.	1884	S.W.bS.	20
Flamborough Head, England, E.	161	N.	4
Formoſa I. China, A.	6670	E.bS.	172
G			
Gambia River, Guinea, Af.	2757	S.S.W.	31
Genoa, Italy, E.	645	S.E.	32
Gibraltar, Spain, E.	1104	S.bW.	20
Glaſgow, Scotland, E.	342	N.W.bN.	11
Gottenburgh, Sweden, E.	642	N.E.	9
Granada I.* Weſt Indies	4405	S.W.bW.	49
Grand Cairo, Egypt, A.	2249	S.E.	45

Diſtance and Bearing from London, &c.

	Diſtance in Engliſh miles.	Bearing.	Time of ſailing in days.
Guernſey I. Britiſh Channel, E.	172	S.W.bS.	4
Gulf of Mexico,* America	4786	W.S.W.	54
H			
Hamburgh, Germany, E.	463	E.N.E.	8
Hanover, Germany, E.	360	E.bN.	—
Havannah,* Cuba I. Weſt Indies	4960	W.S.W.	56
Hiſpaniola I.* W. Indies	4341	S.W.bW.	49
I			
Iceland I. North Sea, E.	1024	N.N.W.	14
Iſpahan, Perſia, A.	2846	E.S.E.	—
J			
Jamaica I.* W. Indies	4765	S.W.bW.	53
Java I. Indian Ocean	7530	S.E.bS.	144
K			
Koningſburgh, Baltic, E.	900	E.bN.	13
L			
Leghorn, Italy, E.	780	S.E.	33
Lima, Peru, S. A.	6450	S.W.	141

Diſtance and Bearing from London, &c.

	Diſtance in Engliſh miles.	Bearing.	Time of ſailing in days.
Liſbon, Portugal, E.	1000	S.W.bS.	14
Liverpool, England, E.	180	N.W.	10
Lizard, England, E.	254	W.S.W.	6
M			
Madeira I.* Atlan, Ocean, Af.	2370	S.W.bS.	28
Madras, E. Indies, A.	5292	S.E.bE.	145
Madrid, Spain, E.	600	S.S.W.	—
Majorca I. Mediterranean Sea, E.	805	S.	26
Maldivia Is. Indian Ocean, A.	5236	S.E.bE.	133
Malta I. Mediterranean Sea, E.	1300	S.E.bE.	39
Manila, Manila I. A.	7163	E.S.E.	167
Mecca, Arabia, A.	3025	S.E.	133
Medina, Arabia, A.	2832	S.E.	—
Mexico, S. A.	6093	W.S.W.	—
Milan, Italy, E.	603	S.E.	—
Minorca I. Mediterranean Sea, E.	725	S.bE.	27
Mocha, Arabia, A.	3619	S.E.	126
N			
Nankin, China, A.	6095	E.bS.	208
Naples, Italy, E.	1000	S.E.	34
Naze * of Norway, E.	513	N.E.bN.	7
Newcaſtle upon Tyne, England, E.	246	N.bW.	5

Diſtance and Bearing from London, &c.

	Diſtance in Engliſh miles.	Bearing.	Time of ſailing in days.
New-York,* N. A.	3590	W.bS.	40
Nile River, A.	2173	S.E.	44
Nubia, Ethiopia, Af.	2915	S.E.bS.	—
P			
Panama, Pacific Ocean, S. A.	5592	S.W.bS.	156
Pekin, China, A.	5510	E.bS.	—
Peterſburgh, Ruſſia, E.	1310	E.N.E.	—
Plymouth, England, E.	218	E.S.E.	5
Point Look out, Greenland, E.	1820	N.N.E.	20
Pondicherry, E. Indies, A.	5267	S.E.bE.	146
Portſmouth, England, E.	76	S.W.	3
Prague, Bohemia, E.	642	E.bS.	—
Q			
Quebec, Canada, N. A.	3270	W.bS.	40
R			
Revel, Ruſſia, E.	1080	E.N.E.	15
Rhodes I. Mediterranean Sea, E.	1673	S.E.bS.	30
Riga, Ruſſia, E.	1050	E.N.E.	15
Rome, Italy, E.	857	S.E.bS.	33

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Diſtance and Bearing from London, &c.

	Diſtance in Engliſh miles.	Bearing.	Time of ſailing in days.
S.			
Sardinia I. Mediterranean Sea, E.	832	S.E.bS.	33
Scaw, Denmark, E.	602	N.E.	7
Shetland Iſ. North Sea, E.	662	N.bW.	9
Siam, Eaſt Indies, A.	6540	E.S.E.	163
Sicily I. Mediterranean Sea, E.	1174	S.E.bS.	32
Smyrna, Turkey, A.	1625	S.E.bE.	41
Sound, entrance into the Baltic	600	N.E.bE.	8
Spitſbergen, Greenland, E.	2032	N.N.E.	24
Stockholm, Sweden, E.	865	N.E.bE.	14
Straits of Babelmandel, entrance into Red Sea, A.	3707	S.E.	124
——— of Magellan, S.A.	8354	S.W.bS.	100
——— Sunda, between the Iſlands Sumatra and Java, Indian Ocean, A.	7521	S.E.bS.	146
Surat, Eaſt Indies, A.	4600	S.E.bE.	138
T			
Teneriff I.* Atlantic Ocean, Af.	1910	S.W.bS.	23
Turin, Italy, E.	585	S.E.bS.	—
V			
Valencia, Spain, E.	855	S.	26

Diſtance and Bearing from London, &c.

	Diſtance in Engliſh miles.	Bearing.	Time of ſailing in days.
Venice, Italy, E.	742	S.E.bE.	40
Vienna, Germany, E.	726	E.S.E.	—
W			
Warſaw, Poland, E.	877	E.	—
Whitby, England, E.	208	N.bW.	4
Whitehaven, England, E.	242	N.W.	11

The time in days multiplied by 80, gives the nautical or geographical miles in the tract of ſailing, thus 145 × 80 = 11600, is the geographical miles in the tract of ſailing from London to Madras.

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THE END.